Homework 5 MATH 1550 Fall 2019

§4.2 #8 Find mean, variance, + std dev of a binomial distribution with $n=316$ and $p=0.82$.

First note $q=1-p=0.18$.

Soln: From the formulas on p.209,

$$\mu = np = (316)(0.82) = 259.12$$

$$\sigma^2 = npq = (316)(0.82)(0.18) = 46.6416,$$

and $\sigma = \sqrt{\sigma^2} = \sqrt{46.6416} \approx 6.8294$

#17 "success" is "too easy to vote" in this problem

So $p=0.27$ is given $\longrightarrow q=1-p=0.73$

$n=12$ was given as the sample size

So the distribution is

$$P(x) = \frac{12!}{(12-x)!\,x!}(0.27)^x(0.73)^{12-x}$$

a) $P(3) = \frac{12!}{9!\,3!}(0.27)^3(0.73)^9 \approx 0.2549$

b) $P(\text{at least four}) = P(4) + P(5) + \dots + P(12)$

easier way

$$= 1 - \left(P(0)+P(1)+P(2)+P(3)\right)$$

$$= 1 - \frac{12!}{12!\,0!}(0.27)^0(0.73)^{12} - \frac{12!}{11!\,1!}(0.27)(0.73)^{11}$$

$$- \frac{12!}{10!\,2!}(0.27)^2(0.73)^{10} - \frac{12!}{9!\,3!}(0.27)^3(0.73)^9$$

$$\approx 0.4137$$

c) $P(\text{less than } 8) = P(0)+P(1)+\cdots+P(7)$

$$\overset{\text{easier way}}{=} 1-\big(P(8)+P(9)+P(10)+P(11)+P(12)\big)$$

$$= \ldots$$

$$\approx 0.995$$

§4.3 #12 This is geometric with $p=\frac{1}{100}=0.01 \longrightarrow q=1-p=0.99$

$$\Rightarrow P(x)=(0.01)(0.99)^{x-1}$$

a) $P(1^{st} \text{ defective is } 10^{th}) = P(10) = (0.01)(0.99)^9$

$$\approx 0.00913$$

(unusual event → probability is < 0.05)

b) $P(1^{st} \text{ defective is } 1^{st}, 2^{nd} \text{ or } 3^{rd})$

$$= P(1)+P(2)+P(3)$$

$$= (0.01)(0.99)^0+(0.01)(0.99)^1+(0.01)(0.99)^2$$

$$= 0.0297$$

c) $P(\text{none of first 10 defective}) = P(11)+P(12)+\cdots$ forever

$$\overset{\text{MUST}}{=} 1-\big(P(1)+\cdots+P(10)\big)$$

$$= \ldots$$

$$\approx 0.9043$$

#13 Given: $\mu = 8 \rightsquigarrow P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{8^x e^{-8}}{x!}$

a) $P(5) = \frac{8^5 e^{-8}}{5!} \approx 0.0916$

infinite-sum

b) $P(\text{at least } 5) = P(5) + P(6) + P(7) + P(8) + \cdots$

necessary trick

$$= 1 - \left(P(0) + P(1) + P(2) + P(3) + P(4)\right)$$

$$= 1 - \frac{8^0 e^{-8}}{0!} - \frac{8^1 e^{-8}}{1!} - \frac{8^2 e^{-8}}{2!} - \frac{8^3 e^{-8}}{3!} - \frac{8^4 e^{-8}}{4!}$$

$$\approx 0.90036$$

Infinite sum

c) $P(\text{more than } 5) = P(6) + P(7) + P(8) + \cdots$

necessary trick

$$= \underbrace{1 - \left(P(0) + P(1) + P(2) + P(3) + P(4) + P(5)\right)}_{\text{this was part (b)}\ldots = 0.90036}$$

$$= 0.90036 - \frac{8^5 e^{-8}}{5!}$$

$$\approx 0.80876$$

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#22) This is a binomial!! Given: $p = 0.29 \rightarrow q = 1 - p = 0.71$
$n = 7$

$$\Rightarrow P(x) = \frac{7!}{(7-x)!\,x!}(0.29)^x(0.71)^{7-x}$$

a) $P(2) = \frac{7!}{5!\,2!}(0.29)^2(0.71)^5 \approx 0.3186$

b) $P(\text{more than } 3) = P(4) + P(5) + P(6) + P(7)$

$$= \frac{7!}{3!\,4!}(0.29)^4(0.71)^3 + \frac{7!}{2!\,5!}(0.29)^5(0.71)^2$$
$$+ \frac{7!}{1!\,6!}(0.29)^6(0.71)^1 + \frac{7!}{0!\,7!}(0.29)^7(0.71)^0$$

≈ 0.133

c) $P\left(\begin{matrix}\text{between } 1+4\\ \text{inclusive}\end{matrix}\right) = P(1) + P(2) + P(3) + P(4)$

$= \ldots$

≈ 0.884

#30) Here $p = 0.005 \longrightarrow q = 0.995$

a) $\mu = \frac{1}{p} = 200$

$\sigma^2 = \frac{q}{p^2} = \frac{0.995}{(0.005)^2} \approx 39800$

$\sigma = \sqrt{\sigma} = \sqrt{39800} \approx 199.4993$

b) it would take about 200 records to find an error

#31) $\mu =$