MATH 1550 - EXAM 3 FALL 2019

SOLUTION

Friday, 22 November
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

Formulas

$E = z_c\sqrt{\dfrac{\hat{p}\hat{q}}{n}}; \hat{p} - E < p < \hat{p} + E$
$t = \dfrac{\bar{x} - \mu}{s_{\bar{x}}}; s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$
$z = \dfrac{\hat{p} - p}{\sigma_{\hat{p}}}; \sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}}$
$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$, d.f.$=n-1$
$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1,\bar{x}_2}}$:   Case I (variances equal): $s_{\bar{x}_1,\bar{x}_2} = \hat{\sigma}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}; \hat{\sigma} = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$;   d.f. $= n_1 + n_2 - 2$ Case II (variances not equal): $s_{\bar{x}_1,\bar{x}_2} = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ d.f. $=$ smaller of $n_1 - 1$ and $n_2 - 1$

Goodness of fit   Observed freq: "O"   expected freq "E"   d.f.$=k-1$, where k is number of categories in the ith category, $E_i = np_i$.   test statistic: $\chi^2 = \sum \dfrac{(O-E)^2}{E}$
ANOVA tests if the means of populations are equal or not; the test statistic uses the F distribution; d.f.$_N = k - 1$ and d.f.$_D = N - k$, where k is the number of samples and N is the sum of the sample sizes

1. In a survey of 1856 WV adults, 421 believe that the Mothman exists. In this problem, you will construct a 90% confidence interval for the population proportion of WV adults who believe the Mothman exists.

   (a) Summarize the data by filling in the table below.

$c =$	0.9
$z_c =$	1.645
$n =$	1856
$\hat{p} =$	0.2268
$\hat{q} =$	0.7732
$E =$	0.01598

   (b) (Fill in the **three** blanks) The confidence interval is

$$0.21082 \leq p \leq 0.24278$$

2. A credit card company claims that the mean credit card debt for individuals is greater than \$5000. You want to test this claim. You find that a random sample of 37 cardholders has a mean credit card balance of \$5122 and a standard deviation of \$625. At $\alpha = 0.05$, can you support the claim?

   (a) Write the hypotheses **and identify which is the claim**.

   $$\begin{cases} H_0: & \mu \leq 5000 \\ \\ H_a: & \mu > 5000 \text{(CLAIM)} \end{cases}$$

   (b) Based on your answer to (a), circle one:

   left-tailed OR right-tailed OR two-tailed

   (c) Compute the test statistic.
   Solution:
   $$s_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{625}{\sqrt{37}} = 102.7493,$$
   and
   $$t = \frac{\overline{x} - \mu}{s_{\overline{x}}} = \frac{5122 - 5000}{102.7493} \approx 1.1873$$

   (d) Circle one:

   reject H_0 OR fail to reject H_0

   (e) Write a sentence to respond to the claim.
   Solution: We fail to reject H_0, meaning there is not enough evidence to support the claim.

3. An auto manufacturer claims that the variance of the gas mileages in a certain vehicle model is 1.0. A random sample of 25 vehicles has a variance of 1.65. Test the claim with $\alpha = 0.05$.

   (a) Write the hypotheses **and identify which is the claim**.

   $$\begin{cases} H_0: & \sigma^2 = 1 \text{(CLAIM)} \\ H_a: & \sigma^2 \neq 1 \end{cases}$$

   (b) Based on your answer to (a), circle one:

   left-tailed OR right-tailed OR two-tailed

   (c) Compute the test statistic.
   Solution:

   $$\chi^2 = \frac{24(1.65)}{1} \approx 39.6$$

   (d) Circle one:

   reject H_0 OR fail to reject H_0

   (e) Write a sentence to respond to the claim.
   Solution: We reject H_0, meaning we do not have evidence to support the claim.

4. A personnel director from Pennsylvania claims that the mean household income is greater in Allegheny County than it is in Erie County. In Allegheny County, a sample of 19 residents has a mean household income of \$49700 and a standard deviation of \$8800. In Erie County, a sample of 15 residents has a mean household income of \$42000 and a standard deviation of \$5100. At $\alpha = 0.05$, can you support the personnel director's claim? Assume the population variances are not equal.

(a) Write the hypotheses **and identify which is the claim**.

$$\begin{cases} H_0: & \mu_1 \leq \mu_2 \\ H_a: & \mu_1 > \mu_2 (\text{CLAIM}) \end{cases}$$

(b) Based on your answer to (a), circle one:

left-tailed OR right-tailed OR two-tailed

(c) Compute the test statistic.

Solution: Since "population variances are not equal",

$$s_{\overline{x_1},\overline{x_2}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{8800^2}{19} + \frac{5100^2}{15}} \approx 2410.3504$$

$$t = \frac{49700 - 42000}{2410.3504} \approx 3.1945$$

(d) Circle one:

reject H_0 OR fail to reject H_0

(e) Write a sentence to respond to the claim.

Solution: Since we reject H_0, there is evidence in support of the claim.

5. A researcher claims that the number of homicide crimes in California by season is distributed according to column 2 in the following table:

Season	Claimed Rate	Actual Measured Frequency
Spring	25%	309
Summer	25%	312
Fall	25%	290
Winter	25%	289

To test the claim, 1200 homicides from a recent year were randomly selected and the season during which each one occurred is recorded in column 3. At $\alpha = 0.05$, test the claim.

(a) Write the hypotheses **and identify which is the claim**.

$$\begin{cases} H_0: & \text{the population distribution obeys teh second column(CLAIM)} \\ \\ H_a: & \text{it doesn't} \end{cases}$$

(b) Compute the test statistic.
Solution:

$$\begin{aligned} \chi^2 &= \sum \frac{(O-E)^2}{E} \\ &= \frac{(309-300)^2}{300} + \frac{(312-300)^2}{300} + \frac{(290-300)^2}{300} + \frac{(289-300)^2}{300} \\ &\approx 1.4866 \end{aligned}$$

(c) Circle one:

reject H_0 OR fail to reject H_0

(d) Write a sentence to respond to the claim.
Solution: Since we fail to reject H_0, there is evidence in support of the claim.

6. The table shows the salaries (in thousands of dollars) for a sample of individuals from the federal, state, and local levels of government. The output of the ANOVA analysis in Excel is pasted next to the table. At $\alpha = 0.01$, can you conclude that at least one mean salary is different from the others?

Federal	State	Local
70.4	52.9	48.8
63.1	37.0	38.3
74.5	54.0	42.6
82.3	54.5	41.0
81.6	56.7	51.6
85.7	61.8	45.7
56.3	39.9	60.3
71.2	50.4	40.8
80.9	53.6	37.2

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Column 1	9	666	74	95.5375
Column 2	9	460.8	51.2	62.495
Column 3	9	406.3	45.1444	54.6903

ANOVA

ce of Varia	*SS*	*df*	*MS*	*F*
Between G	4167.46	2	2083.73	29.3865
Within Grc	1701.78	24	70.9076	
Total	5869.24	26		

(a) Write the hypotheses **and identify which is the claim**.

$$\begin{cases} H_0: & \mu_1 = \mu_2 = \mu_3 \\ H_a: & \text{at least one is not equal to the others (CLAIM)} \end{cases}$$

(b) Compute the test statistic.
Solution: Reading from the ANOVA chart,

$$F = 29.3865.$$

(c) Circle one:

reject H_0 OR fail to reject H_0

(d) Write a sentence to respond to the claim.
Solution: Since we reject H_0, there is evidence to support the claim.

Table 7—*F*-Distribution *(continued)*

d.f.$_D$: Degrees of freedom, denominator	$\alpha = 0.01$																		
	d.f.$_N$: Degrees of freedom, numerator																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**∞**
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	4.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00