MATH 1550 - EXAM 2 FALL 2019

SOLUTION

Friday, 18 October
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

Binomial: $P(x) = \dfrac{n!}{(n-x)!x!}p^x q^{n-x}$
Geometric: $P(x) = pq^{x-1}$
Poisson: $P(x) = \dfrac{\mu^x e^{-\mu}}{x!}$
$E = z_c \dfrac{\sigma}{\sqrt{n}}; \bar{x} - E < \mu < \bar{x} + E$
$E = t_c \dfrac{s}{\sqrt{n}}; \bar{x} - E < \mu < \bar{x} + E$

$\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
$z = \dfrac{x-\mu}{\sigma}$
$z = \dfrac{\bar{x}-\mu}{\sigma_{\bar{x}}}; \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
$x = z\sigma + \mu$

1. (12 points) An auto parts seller finds that 1 in every 200 parts sold is defective. Find the probability that the first defective part is the fifth part sold.
   Solution: This is a geometric distribution with $p = \frac{1}{200} = 0.005$ and hence $q = 1 - p = 0.995$. It has probability mass function
   $$P(x) = 0.005(0.995)^{x-1}.$$
   Therefore, the probability that the first defective part is the fifth part sold is given by
   $$P(5) = 0.005(0.995)^{5-1} = 0.005(0.995)^4 = 0.004900.$$

2. (12 points) Forty-three percent of U.S. adults think that the government should help fight childhood obesity. You randomly select five U.S. adults. Find the probability that the number of U.S. adults who think that the government should fight childhood obesity is exactly two.
   Solution: This is a binomial distribution with $n = 5$, $p = 0.43$, and $q = 1 - p = 0.57$. It has probability mass function
   $$P(x) = \frac{5!}{(5-x)!x!} 0.43^x 0.57^{5-x}.$$
   Therefore, the probability that the number of U.S. adults who think that the government should fight childhood obesity is exactly two is given by
   $$P(2) = \frac{5!}{(5-2)!2!} 0.43^2 0.57^{5-2} = \frac{5!}{3!2!} 0.43^2 0.57^3 = 0.3424$$

3. (12 points) The mean number of heart transplants performed per day in the U.S. in a recent year was about eight. Find the probability that the number of heart transplants performed on any given day is no more than 1.
   Solution: This is a Poisson distribution with $\mu = 8$. It has probability mass function
   $$P(x) = \frac{8^x e^{-8}}{x!}.$$
   Therefore, the probability that the number of heart transplants performed on any given day is no more than 1 is given by
   $$P(x \leq 1) = P(0) + P(1) = \frac{8^0 e^{-8}}{0!} + \frac{8^1 e^{-8}}{1!} = 0.0030191.$$

4. (16 points) The monthly utility bills in a city are normally distributed, with a mean of \$100 and a standard deviation of \$12. Find the probability that a randomly selected utility bill is between \$90 and \$120.
   Solution: Take the given range
   $$\$90 \leq x \leq \$120$$
   Substract $\mu = \$100$ and divide by $\sigma = \$12$ to get
   $$\frac{\$90 - \$100}{\$12} \leq \frac{x - \mu}{\sigma} \leq \frac{\$120 - \$100}{\$12}$$
   and simplify to get
   $$-0.83 \leq z \leq 1.66.$$
   We can now look this probability up in the normal table:
   $$P(-0.83 \leq z \leq 1.66) = P(z \leq 1.66) - P(z \leq -0.83) = 0.9515 - 0.2033 = 0.7482.$$

5. (16 points) The time spent (in days) waiting for a kidney transplant for people ages 35-49 can be approximated by a normal distribution with mean 1674 and standard deviation 212.5. What waiting time represented the 60th percentile?
Solution: We reverse lookup $P_{60} = 0.6$ in the normal distribution table and see that it corresponds to $z = 0.255$. Using the formula $x = z\sigma + \mu$ we obtain

$$x = 0.255(212.5) + 1674 = 1728.1875.$$

6. (16 points) As reported by the website `westvirginiagasprices.com`, the mean price of gasoline in West Virginia on 14 October 2019 was \$2.553 per gallon. A random sample of 38 gas stations is selected from this population. What is the probability that the mean price from the sample was less than \$2.50 on that day? Assume that $\sigma = \$0.61$.
Solution: Here $\mu_{\overline{x}} = \mu = 2.553$ and $\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{0.61}{\sqrt{38}} \approx 0.09895$. We want to know what $P(x \leq 2.50)$ is, so we will normalize it by subtracting $\mu_{\overline{x}}$ and dividing by $\sigma_{\overline{x}}$ to get

$$\frac{x - \mu_{\overline{x}}}{\sigma_{\overline{x}}} \leq \frac{2.50 - 2.553}{0.09895},$$

hence

$$z \leq -0.53.$$

Therefore,

$$P(x \leq 2.50) = P(z \leq -0.53) = 0.2981.$$

7. (16 points) In a random sample of five people, the mean driving distance to work was 15.3 miles and the sample standard deviation was 3.2 miles. Construct a 95% confidence interval for the population mean.
   (a) (8 points) Summarize the data by filling out the table below:

c	$= 0.95$
n	$=5$
d.f.	$= 4$
(Critical value) t_c	$= 2.776$
$\bar{x}$	$=15.3$
s	$=3.2$
E	$= t_n \dfrac{s}{\sqrt{n}} = 2.776\left(\dfrac{3.2}{\sqrt{5}}\right) = 3.9726$

   (b) (8 points) (Fill in the **three** blanks) The confidence interval is:

$$11.3274 < \mu < 19.2726$$