MATH 1550 - EXAM 1 FALL 2019 SOLUTION

Wednesday, 18 September
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

Formulas

range = max − min

class width = $\dfrac{\text{range}}{\text{number of classes}}$

$\mu = \overline{x} = \dfrac{\sum x}{N}$

$\sigma^2 = \dfrac{\sum (x-\mu)^2}{N}$

$\sigma = \sqrt{\sigma^2} = \sqrt{\dfrac{\sum (x-\mu)^2}{N}}$

$s^2 = \dfrac{\sum (x-\mu)^2}{N-1}$

$s = \sqrt{s^2} = \sqrt{\dfrac{\sum (x-\mu)^2}{N-1}}$

multiplication rule: $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$

addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

z-score: $z = \dfrac{x-\mu}{\sigma}$

Empirical Rule (for bell-shaped data sets)

1. 65% of the data lies within 1 standard deviation of the mean
2. 95% of the data lies within 2 standard deviations of the mean
3. 99.7% of the data lies within 3 standard deviations of the mean

Chebyshev theorem (for any shaped data sets)

The proportion of the data lying k standard deviations from the mean is at least $1 - \dfrac{1}{k^2}$.

1. (6 points) Consider the following data representing daily saturated fat intakes (in grams) of 20 people:

38 32 34 39 40 54 32 17 29 33 57 40 25 36 33 24 42 16 31 33

Construct a relative frequency distribution for the data using five classes.
Solution:

$$\text{range=max-min=57-16=41}$$

$$\text{class width} = \frac{\text{range}}{\#\text{ classes}} = \frac{31}{5} = 8.2$$

Class	Frequency	Relative Frequency
16–24.2	3	$\frac{3}{20}$
24.2–32.4	5	$\frac{5}{20}$
32.4–40.6	9	$\frac{9}{20}$
40.6 − 48.8	1	$\frac{1}{20}$
48.8 − 57	2	$\frac{2}{20}$

2. (12 points) Consider the following data representing the number of credits taken by a sample of thirteen college students:

12 14 16 15 13 14 15 18 16 16 12 16 16

(a) (6 points) Find the median.
Solution: Order the data:

12 12 13 14 14 15 (15) 16 16 16 16 16 18

(b) (6 points) Find the mode.
Solution: The mode occurs most often. In this case, it is 16.

3. (14 points) The total ballots cast in West Virginia general elections (in hundreds of thousands) between 2010 and 2018 are given by

535.1 685.0 462.8 732.3 597.1

(a) (7 points) Find the mean of this data.
Solution: Calculate

$$\mu = \frac{535.1 + 685.0 + 462.8 + 732.3 + 597.1}{5} = 602.46$$

(b) (7 points) Find the population standard deviation of this data.
Solution: Compute

$$\begin{aligned}\sigma^2 &= \frac{(535.1 - 602.46)^2 + (685.0 - 602.46)^2 + (462.8 - 602.46)^2 + (732.3 - 602.46)^2 + (597.1 - 602.46)^2}{5} \\ &= 9548.4584,\end{aligned}$$

so the standard deviation is

$$\sigma = \sqrt{\sigma^2} = \sqrt{9548.4584} = 97.7162.$$

4. (14 points) (a) (7 points) The mean monthly utility bill for a sample of households is \$82 with a standard deviation of \$6. Between what two values does approximately 95% of the data lie? (Assume the data has a bell-shaped distribution).
Solution: Here $\overline{x} = 86$ and $s = 6$. Since the empirical rule says that 95% of the data lies within two standard deviations of the mean, we compute that:

$$\$70 \xleftarrow{-\$6} \$76 \xleftarrow{-\$6} \$82 \xrightarrow{+\$6} \$88 \xrightarrow{+\$6} \$94,$$

and so we know that about 95% of the monthly utility bills were between \$70 and \$94.

(b) (7 points) Old Faithful is a geyser in Yellowstone National Park. From a sample with $n = 32$, the mean duration of Old faithful's eruptions is 3.32 minutes and the standard deviation is 1.09 minutes. Using Chebyshev's theorem, determine at least how many of the eruptions lasted between 1.14 minutes and 5.5 minutes.
Solution: We need to determine how many standard deviations $s = 1.09$ away from the mean $\overline{x} = 3.32$ that 1.14 and 5.5 are. So,

$$1.14 \xleftarrow{-1.09} 2.23 \xleftarrow{-1.09} 3.32 \xrightarrow{+1.09} 4.41 \xrightarrow{+1.09} 5.5$$

So we see that 1.14 and 5.5 are within two standard deviations from the mean. This means we will take $k = 2$ and we get that at least

$$1 - \frac{1}{k^2} = 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75,$$

of the eruptions lasted between 1.14 and 5.5 minutes. If we compute 75% of 32, we conclude that at least $0.75(32) = 24$ of the eruptions in the data set lasted between these times.

5. (12 points) Consider the data

$$4 \quad 17 \quad 7 \quad 14 \quad 18 \quad 12 \quad 3 \quad 16 \quad 10 \quad 4 \quad 4 \quad 11$$

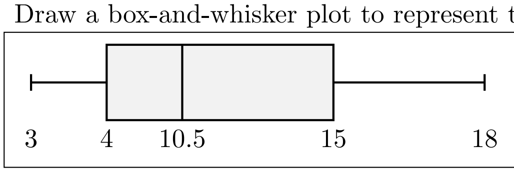
(a) (6 points) Find the quartiles Q_1, Q_2, and Q_3.
Solution: First order the data:

$$3 \quad 4 \quad 4 \quad 4 \quad 7 \quad 10 \quad 11 \quad 12 \quad 14 \quad 16 \quad 17 \quad 18$$

Since the middle two data points are 10 and 11, the median is the average: $\frac{10+11}{2} = 10.5$ and so we have $Q_2 = 10.5$. For Q_1 there are two middle data points, but both are 4 so we have $Q_1 = 4$. For Q_3, the middle data points are 14 and 16, hence we take the average to get $Q_3 = \frac{14+16}{2} = 15$.

(b) (6 points) Draw a box-and-whisker plot to represent the data.

Solution:



6. (20 points) A probability experiment is conducted where two coins and two 3-sided dice (i.e. sides labelled “1”, “2”, and “3”) are rolled, in that order.

   (a) (6 points) How many outcomes are in the sample space?
   Solution: 36

   (b) (7 points) How many outcomes are in the event described as “all coin flips are heads and the sum of the die rolls is 4”?
   Solution: 3

   (c) (7 points) How many outcomes are in the event described as “the first coin flip is heads and the first die roll is bigger than 1”? What is the probability of this event?
   Solution: number of outcomes: 12; the probability is $\frac{12}{36} = \frac{1}{3}$

7. (11 points) A statistics class has 21 students. Of these, 11 are sophomores and 8 are math majors. Of the math majors, 2 are sophomores. What is the probability that a randomly selected student is a sophomore or is a math major?
   Solution: We are told that $P(\text{sophomore}) = \frac{11}{21}$, $P(\text{math major}) = \frac{8}{21}$, and $P(\text{sophomore}|\text{math major}) = \frac{2}{8}$. Using the addition rule,

$$P(\text{sophomore and math major}) = P(\text{math major})P(\text{sophomore}|\text{math major}) = \frac{8}{21} \cdot \frac{2}{8} = \frac{2}{21}.$$

   We may now calculate

$$\begin{aligned} P(\text{sophomore or math}) &= P(\text{sophomore}) + P(\text{math major}) - P(\text{sophomore and math major}) \\ &= \frac{11}{21} + \frac{8}{21} - \frac{2}{21} \\ &= 0.80952. \end{aligned}$$

8. (11 points) Of the video cards manufactured by a company, 1% have dead pins, 6% have some faulty transistors, and 0.3% have both dead pins and faulty transistors. Find the probability that a randomly selected video card has a dead pin or a faulty transistor.
   Solution: We are told that $P(\text{dead pins}) = 0.01$, $P(\text{faulty transistor}) = 0.06$ and $P(\text{dead pins and faulty transistor}) = 0.003$. Therefore using the addition rule, we compute

$$\begin{aligned} P(\text{dead pin or faulty transistor}) &= P(\text{dead pin}) + P(\text{faulty transistor}) - P(\text{dead pin and faulty transistor}) \\ &= 0.01 + 0.06 - 0.003 \\ &= 0.067. \end{aligned}$$