Homework 9 MATH 1540 Fall 2019

§9.2 #5 $\cos\left(\frac{\pi}{12}\right)$

Soln: Find a way to write $\frac{\pi}{12}$ using angles on unit circle:

$$\frac{\pi}{4}-\frac{\pi}{6} = \frac{3\pi}{12}-\frac{2\pi}{12} = \frac{\pi}{12} \checkmark$$

So,

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$

$$\overset{\text{identity}}{=} \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

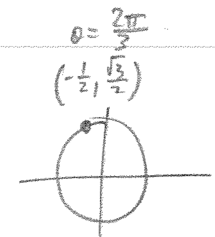
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

#13

$$\cos\left(x+\frac{2\pi}{3}\right) \underset{\substack{\uparrow \\ \text{identity}}}{=} \cos(x)\cos\left(\frac{2\pi}{3}\right)-\sin(x)\sin\left(\frac{2\pi}{3}\right)$$

$$= -\frac{1}{2}\cos(x)-\frac{\sqrt{3}}{2}\sin(x)$$



#16

$$\cot\left(\frac{\pi}{2}-x\right) = \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)} \underset{\substack{\uparrow \\ \text{identities}}}{=} \frac{\overbrace{\cos\left(\frac{\pi}{2}\right)}^{=0}\cos(x)+\overbrace{\sin\left(\frac{\pi}{2}\right)}^{=1}\sin(x)}{\sin\left(\frac{\pi}{2}\right)\cos(x)-\cos\left(\frac{\pi}{2}\right)\sin(x)}$$

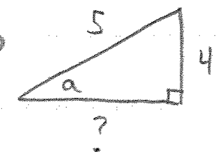
$$= \frac{0+\sin(x)}{\cos(x)-0}$$

$$= \frac{\sin(x)}{\cos(x)}$$

$$= \tan(x)$$

#21) Given $\sin(a) = \frac{4}{5}$ and $\cos(b) = \frac{1}{3}$, find $\sin(a-b)$ and $\cos(a+b)$. 2

Soln: First $\sin(a) = \frac{4}{5}$ $\Rightarrow$



$\Rightarrow$ $?^2 + 4^2 = 5^2$

$?^2 + 16 = 25$

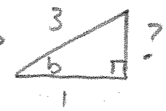
$?^2 = 9$

$? = 3$

$\Rightarrow \cos(a) = \frac{3}{5}$

Similarly,

$\cos(b) = \frac{1}{3}$ $\Rightarrow$



$\Rightarrow$ $1^2 + ?^2 = 3^2$

$? = \sqrt{8}$

$\Rightarrow \sin(b) = \frac{\sqrt{8}}{3}$

Therefore,

$$\sin(a-b) \underset{\uparrow \text{ identity}}{=} \sin(a)\cos(b) - \cos(a)\sin(b)$$

$$= \left(\frac{4}{5}\right)\left(\frac{1}{3}\right) - \left(\frac{3}{5}\right)\left(\frac{\sqrt{8}}{3}\right)$$

$$= \frac{4}{15} - \frac{3\sqrt{8}}{15}$$

and

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$= \left(\frac{3}{5}\right)\left(\frac{1}{3}\right) - \left(\frac{4}{5}\right)\left(\frac{\sqrt{8}}{3}\right)$$

$$= \frac{3 - 4\sqrt{8}}{15}$$

#241 $\tan\left(\sin^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{1}{2}\right)\right) = \dfrac{\sin\left(\sin^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{1}{2}\right)\right)}{\cos\left(\sin^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{1}{2}\right)\right)}$

identities

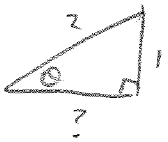
$$= \frac{\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)-\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)}{\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)+\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)}$$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\downarrow$

$\sin\theta = \frac{1}{2}$

$\downarrow$



$\Rightarrow ?^2 + 1^2 = 2^2$

$? = \sqrt{3}$

$\downarrow$

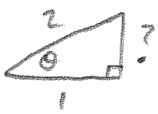
$\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \cos(\theta) = \frac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\downarrow$

$\cos(\theta) = \frac{1}{2}$

$\downarrow$



$\Downarrow$

$1^2 + ?^2 = 2^2$

$? = \sqrt{3}$

$\downarrow$

$\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \sin(\theta) = \frac{\sqrt{3}}{2}$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}}{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{\frac{1}{4}-\frac{3}{4}}{\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}}$$

$$= \frac{-2/4}{2\sqrt{3}/4} = \frac{-2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$$