

#6) Solve for  $\theta$  in  $[0, 2\pi)$ :

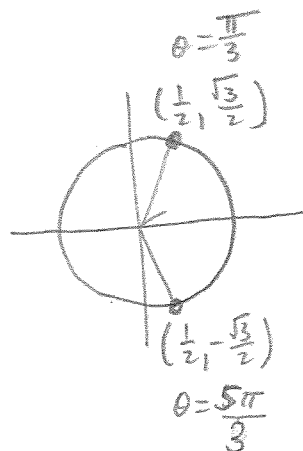
$$2\cos(\theta) = 1$$

Soln: Div by 2,

$$\cos(\theta) = \frac{1}{2}$$

↓ from unit circle

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$



#17) Solve for  $\theta$  in  $[0, 2\pi)$ :  $2\sin(3\theta) = 1$

Soln: Divide by 2 to get

$$\sin(3\theta) = \frac{1}{2}$$

Let  $\psi = 3\theta$ , so it becomes

$$\sin(\psi) = \frac{1}{2}$$

↓ unit circle

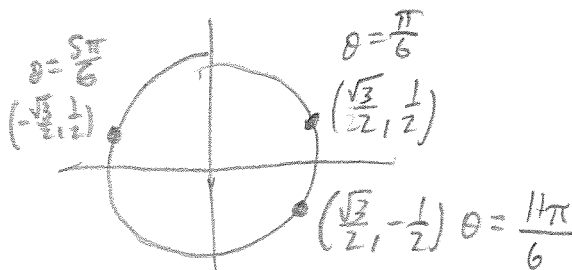
$$\psi = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \begin{cases} 3\theta = \psi = \frac{\pi}{6} + 2n\pi \\ 3\theta = \psi = \frac{5\pi}{6} + 2n\pi \end{cases}$$

$$\Rightarrow \begin{cases} \theta = \frac{\pi}{18} + \frac{2n\pi}{3} = \frac{\pi + 12n\pi}{18} & (i) \\ \theta = \frac{5\pi}{18} + \frac{2n\pi}{3} = \frac{5\pi + 12n\pi}{18} & (ii) \end{cases}$$

We need to find which  $n$  cause  $\theta$  in  $[0, 2\pi)$ ,

i.e.,  $0 \leq \theta < 2\pi$ , or

$$0 \leq \theta < \frac{32\pi}{18}$$



For (i)

$$\begin{aligned}
 n=3: \theta &= \frac{\pi + 36\pi}{18} = \frac{37\pi}{18} \times \\
 n=2: \theta &= \frac{\pi + 24\pi}{18} = \frac{25\pi}{18} \checkmark \\
 n=1: \theta &= \frac{\pi + 12\pi}{18} = \frac{13\pi}{18} \checkmark \\
 n=0: \theta &= \frac{\pi}{18} \checkmark \\
 n=-1: \theta &= \frac{\pi - 12\pi}{18} = \frac{-11\pi}{18} \times
 \end{aligned}$$

} good

$$\theta = \frac{\pi + 12n\pi}{18} \rightsquigarrow$$

$$\begin{aligned}
 \theta &= \frac{5\pi + 12n\pi}{18} \rightarrow \\
 n=2: & \frac{29\pi}{18} \checkmark & n=3: & \frac{41\pi}{18} \times \\
 n=1: & \frac{17\pi}{18} \checkmark \\
 n=0: & \frac{5\pi}{18} \checkmark \\
 n=-1: & -\frac{7\pi}{18} \times
 \end{aligned}$$

Therefore the solutions are

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$$

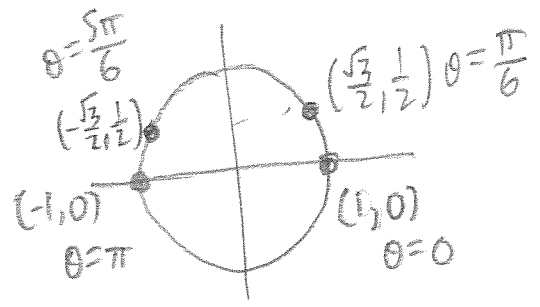
#24) Solve for  $\theta$  in  $[0, 2\pi)$ :  $\tan(x) - 2\sin(x)\tan(x) = 0$

Soln: Factor the  $\tan(x)$  to get

$$\tan(x) [1 - 2\sin(x)] = 0$$

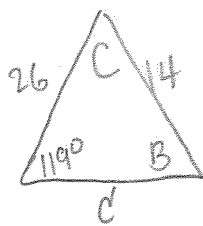
$$\begin{aligned}
 \tan(x) &= 0 \\
 \downarrow \\
 x &= 0, \pi
 \end{aligned}$$

$$\begin{aligned}
 1 - 2\sin(x) &= 0 \\
 \sin(x) &= \frac{1}{2} \\
 \downarrow \\
 x &= \frac{\pi}{6}, \frac{5\pi}{6}
 \end{aligned}$$



§10.1 | #14 |  $A=119^\circ$ ,  $a=14$ ,  $b=26$

(3)



Soln: Find B

$$\frac{\sin(B)}{26} = \frac{\sin(119^\circ)}{14}$$

↓ mult by 26

$$\sin(B) = \frac{26 \sin(119^\circ)}{14}$$

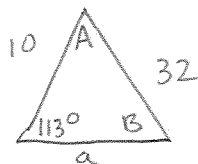
↓ take arcsin

calculator in DEGREE mode

$$B = \sin^{-1}\left(\frac{26 \sin(119^\circ)}{14}\right) \approx \text{ERROR!!}$$

So this "Δ" does not exist!!

#15 |  $C=113^\circ$ ,  $b=10$ ,  $c=32$



Soln: Find B

$$\frac{\sin(B)}{10} = \frac{\sin(113^\circ)}{32}$$

↓ mult by 10

$$\sin(B) = \frac{10 \sin(113^\circ)}{32}$$

↓ take arcsin

$$B = \sin^{-1}\left(\frac{10 \sin(113^\circ)}{32}\right) \approx 16.72^\circ$$

Second solution?

$$B_{\text{Second}} = 180^\circ - 16.72^\circ$$

$$= 163.28^\circ$$

Too big because known angle is  $113^\circ$ .

ONLY one solution!

Find A

$$119^\circ + 16.72^\circ + C = 180^\circ$$

$$C = 180^\circ - 119^\circ - 16.72^\circ = 44.28^\circ$$

Find a

$$\frac{\sin(44.28^\circ)}{a} = \frac{\sin(119^\circ)}{14}$$

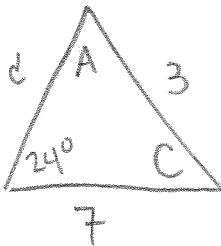
Multiply by a to get

$$\sin(44.28^\circ) = a \frac{\sin(119^\circ)}{14}$$

Mult. by  $\frac{14}{\sin(119^\circ)}$  to get

$$a = \frac{14 \sin(44.28^\circ)}{\sin(119^\circ)} \approx 11.175$$

#20 Solve



Soln: Find A

$$\frac{\sin(24^\circ)}{3} = \frac{\sin(A)}{7}$$

↓ mult by 7

$$\sin(A) = \frac{7 \sin(24^\circ)}{3}$$

↓

$$A = \sin^{-1}\left(\frac{7 \sin(24^\circ)}{3}\right) \approx 71.632$$

Second solution?

$$A = 180^\circ - 71.632^\circ \\ = 108.368^\circ$$

Small enough!

So we get two solutions

Two solutions:

$$A = 71.632^\circ$$

Find C

$$71.632^\circ + 24^\circ + C = 180^\circ$$



$$C = 180^\circ - 71.632^\circ - 24^\circ \\ = 84.368^\circ$$

Find c

$$\frac{\sin(84.368^\circ)}{c} = \frac{\sin(24^\circ)}{3}$$



$$c = \frac{3 \sin(84.368^\circ)}{\sin(24^\circ)}$$

$$\approx 7.340$$

$$A = 108.368^\circ$$

Find C

$$108.368^\circ + 24^\circ + C = 180^\circ$$

$$C = 180^\circ - 108.368^\circ - 24^\circ \\ = 47.632^\circ$$

Find c

$$\frac{\sin(47.632^\circ)}{c} = \frac{\sin(24^\circ)}{3}$$



$$c = \frac{3 \sin(47.632^\circ)}{\sin(24^\circ)}$$

$$\approx 5.449$$