§9.2 #49 Prove $\frac{\cos(a+b)}{\cos(a)\cos(b)} = 1-\tan(a)\tan(b)$

Soln: Start w/ left;

$$\frac{\cos(a+b)}{\cos(a)\cos(b)} \underset{\uparrow \text{ identity}}{=} \frac{\cos(a)\cos(b)-\sin(a)\sin(b)}{\cos(a)\cos(b)}$$

$$= \frac{\cancel{\cos(a)}\cancel{\cos(b)}}{\cancel{\cos(a)}\cancel{\cos(b)}} - \frac{\sin(a)}{\cos(a)}\frac{\sin(b)}{\cos(b)}$$

tan(a) → tan(b)

$$= 1 - \tan(a)\tan(b),$$

as was to be shown.

#50 Prove $\cos(x+y)\cos(x-y) = \cos^2(x)-\sin^2(y)$

Soln: Start w/ left,

$$\cos(x+y)\cos(x-y) = [\cos(x)\cos(y)-\sin(x)\sin(y)][\cos(x)\cos(y)+\sin(x)\sin(y)]$$

$$= \cos^2(x)\cos^2(y) + \cancel{\cos(x)\cos(y)\sin(x)\sin(y)}$$
$$- \cancel{\sin(x)\sin(y)\cos(x)\cos(y)} - \sin^2(x)\sin^2(y)$$

$\sin^2x+\cos^2x=1$
$\downarrow$
$\sin^2(x)=1-\cos^2x$

$$= \underbrace{\cos^2(x)}_{\text{want}}\underbrace{\cos^2(y)}_{\text{don't want}} - \underbrace{\sin^2(x)}_{\text{don't want}}\underbrace{\sin^2(y)}_{\text{want}}$$

$\sin^2y+\cos^2y=1$
$\downarrow$
$\cos^2y=1-\sin^2y$

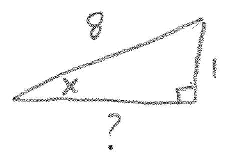
$$= \cos^2(x)(1-\sin^2(y)) - \sin^2(y)(1-\cos^2(x))$$

$$= \cos^2(x) - \cancel{\cos^2(x)\sin^2(y)} - \sin^2(y) + \cancel{\sin^2(y)\cos^2(x)}$$

$$= \cos^2(x) - \sin^2(y)$$

§9.3 | #5 | if $\sin(x) = \frac{1}{8}$ and x in QI, then find $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$

Soln: From $\sin(x) = \frac{1}{8}$, draw a Δ:



$\rightarrow\ ?^2 + 1^2 = 8^2$

$\rightarrow\ ? = \sqrt{63}$

Since x in QII,

$$\cos(x) = -\frac{\sqrt{63}}{8}$$

Therefore,

$$\sin(2x) = 2\sin(x)\cos(x) = 2\left(\frac{1}{8}\right)\left(-\frac{\sqrt{63}}{8}\right)$$

$$= \frac{-2\sqrt{63}}{64} = \frac{-\sqrt{63}}{32}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$= \left(-\frac{\sqrt{63}}{8}\right)^2 - \left(\frac{1}{8}\right)^2 = \frac{63}{64} - \frac{1}{64} = \frac{62}{64} = \frac{31}{32}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{-\frac{\sqrt{63}}{32}}{\frac{31}{32}} = -\frac{\sqrt{63}}{31}$$

#14 | $\cos\left(-\frac{11\pi}{12}\right)$

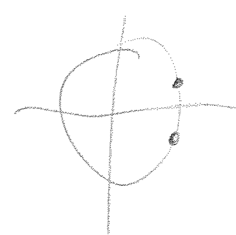
Soln: $-\frac{11\pi}{12} = \frac{\alpha}{2} \rightarrow \alpha = -\frac{11\pi}{6}$

identify

Therefore, $\cos\left(-\frac{11\pi}{12}\right) = \cos\left(\frac{-\frac{11\pi}{6}}{2}\right) = +\sqrt{\frac{1+\cos\left(-\frac{11\pi}{6}\right)}{2}}$

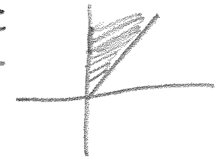
in QI

$$= \sqrt{\frac{1+\sqrt{3}/2}{2}}$$



#22) if $\csc(x)=7$ and x in QII, then find $\sin(\frac{x}{2})$, $\cos(\frac{x}{2})$, and $\tan(\frac{x}{2})$

Soln: if x in QII, then $\frac{\pi}{2} < x < \pi \Rightarrow \underbrace{\frac{\pi}{4} < x < \frac{\pi}{2}}_{\text{in QI}}$

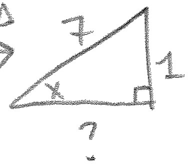


y all in QI ⇒ all (+)

$\frac{1}{\sin(x)} = \csc(x) = 7$

$\downarrow$

$\sin(x) = \frac{1}{7} \xrightarrow{\text{draw }\Delta}$



$\Rightarrow ?^2 + 1^2 = 7^2 \longrightarrow \cos(x) = \frac{\sqrt{48}}{7}$

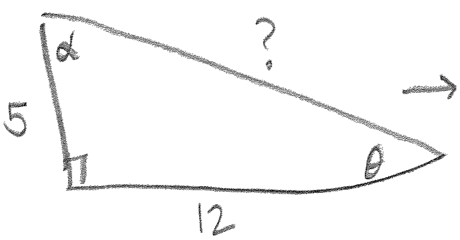
$? = \sqrt{48}$

$$\sin\left(\frac{x}{2}\right) = +\sqrt{\frac{1-\cos(x)}{2}} = \sqrt{\frac{1-\frac{\sqrt{48}}{7}}{2}}$$

$$\cos\left(\frac{x}{2}\right) = +\sqrt{\frac{1+\cos(x)}{2}} = \sqrt{\frac{1+\frac{\sqrt{48}}{7}}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})} = \frac{\sqrt{\frac{1-\sqrt{48}/7}{2}}}{\sqrt{\frac{1+\sqrt{48}/7}{2}}}$$

#25)



$\longrightarrow 12^2 + 5^2 = ?^2$

$144 + 25 = ?^2$

$? = \sqrt{169} = 13$

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{13^2}$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$\tan(2\alpha) = \frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{\frac{120}{13^2}}{\frac{5^2-12^2}{13^2}} = \frac{120}{119}$$

3 14
~~144~~
25
119

25-144

#36) Prove identity

$$\cot(x) - \tan(x) = 2\cot(2x)$$

Soln: Start with right:

$$2\cot(2x) = \frac{2\cos(2x)}{\sin(2x)}$$

$$= \cancel{2}\left[\frac{\cos^2(x) - \sin^2(x)}{\cancel{2}\cos(x)\sin(x)}\right]$$

$$= \frac{\cos^{\cancel{2}}(x)}{\cancel{\cos(x)}\sin(x)} - \frac{\sin^{\cancel{2}}(x)}{\cos(x)\cancel{\sin(x)}}$$

$$= \frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)}$$

$= \cot(x) - \tan(x)$, as was to be shown. ∎

#58) Prove identity

$$(\sin^2(x) - 1)^2 = \cos(2x) + \sin^4(x)$$

Soln: From left,

$$(\sin^2(x) - 1)^2 = \sin^4(x) \underbrace{- 2\sin^2(x) + 1}_{= \cos(2x) \text{ by identity}}$$

$$= \sin^4(x) + \cos(2x),$$

as was to be shown. ∎