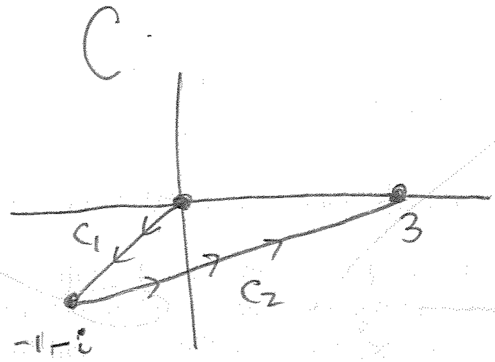
Quiz 4 MATH 1199 Fall 2019



Calculate $\displaystyle\int_C \underbrace{Re(z)}_{f(z)}\, dz$

Soln:

Parametrize C_1

$$\begin{cases} z(t) = t(-1-i) \\ 0 \le t \le 1 \end{cases}$$

$$\downarrow$$

$$z'(t) = -1-i$$

$$f(z(t)) = Re(-t-ti) = -t$$

Parametrize C_2

$$\begin{cases} z(t) = 3t + (-1-i)(1-t) = \\ 0 \le t \le 1 \end{cases}$$

$$\downarrow$$

$$z'(t) = 3 + (-1)(-1-i) = 4+i$$

$$f(z(t)) = Re\big(3t + (1-t)(-1-i)\big)$$
$$= Re(3t - 1 - i + t + ti)$$
$$= Re\big((4t-1) + (t-1)i\big)$$
$$= 4t-1$$

So,

$$\int_C Re(z)\,dz = \int_{C_1} Re(z)\,dz + \int_{C_2} Re(z)\,dz$$
$$= \int_0^1 (-t)(-1-i)\,dt + \int_0^1 (4t-1)(4+i)\,dt$$
$$= (1+i)\int_0^1 t\,dt + (4+i)\int_0^1 4t-1\,dt$$
$$= (1+i)\left[\frac{t^2}{2}\Big|_0^1 + (4+i)\left[2t^2 - t\Big|_0^1\right.\right.$$
$$= (1+i)\left[\frac{1}{2} - 0\right] + (4+i)\left[(2-1) - 0\right]$$
$$= \frac{1+i}{2} + (4+i)(1)$$
$$= \frac{3}{2}i + \frac{9}{2}$$