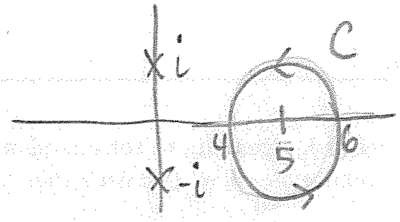
HW9 MATH 1199 Fall 2019

CIF

$$f(y) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-y}\,dz,$$

where C has y in interior

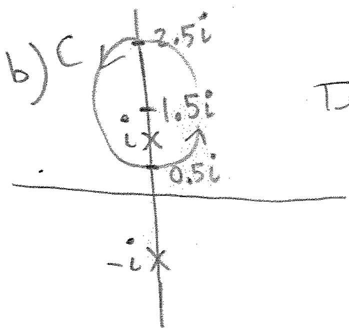
① a) blow-up points : $\pm i$



Thus by Cauchy theorem,

$$\int_C \frac{\sin(z)\cos(z)}{z^2+1}\,dz = 0$$

b)



Therefore, $z=i$ is only blow-up point interior to C.

Write

$$\frac{\sin(z)\cos(z)}{z^2+1} = \frac{\dfrac{\sin(z)\cos(z)}{z+i}}{z-i} \quad \leftarrow \text{"}f(z)\text{"}$$

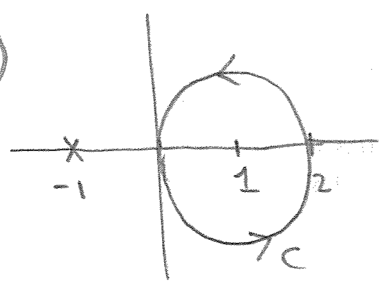
By Cauchy integral formula,

$$\int_C \frac{\sin(z)\cos(z)}{z^2+1}\,dz = \int_C \frac{\dfrac{\sin(z)\cos(z)}{z+i}}{z-i}\,dz$$

$$= \frac{2\pi \cancel{i}\sin(i)\cos(i)}{2\cancel{i}}$$

$$= \pi\sin(i)\cos(i)$$

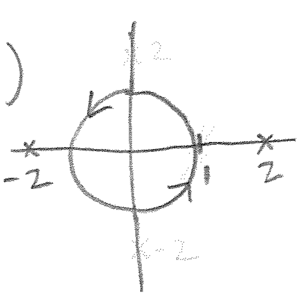
c)



Thus by Cauchy theorem,

$$\int_C \frac{e^z\sin(z)}{z+1}\,dz = 0.$$

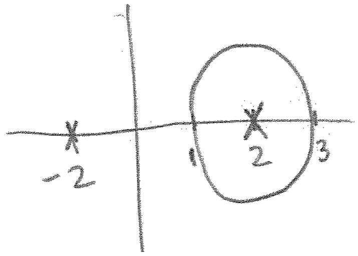
d)



Thus by Cauchy theorem,

$$\int_C \frac{z^3+5z^2-2z+1}{z^2-4}\,dz = 0$$

e)



Therefore, $z=2$ is only blow-up point interior to C.

So, compute

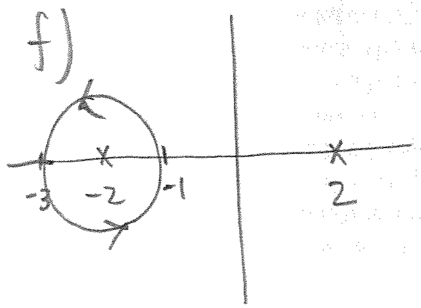
$$\int_C \frac{z^3+5z^2-2z+1}{z^2-4}\,dz = \int_C \frac{\frac{z^3+5z^2-2z+1}{z+2}}{z-2}\,dz$$

Cauchy integral formula

$$= (2\pi i)\left(\frac{2^3+5\cdot 2^2-2(2)+1}{2+2}\right)$$

$$= \frac{50\pi i}{4} = \frac{25\pi i}{2}$$

f)



Therefore, $z=-2$ is only blow-up point interior to C. So, compute

$$\int_C \frac{z^3+5z^2-2z+1}{z^2-4}\,dz = \int_C \frac{\frac{z^3+5z^2-2z+1}{z-2}}{z+2}\,dz$$

Cauchy integral formula

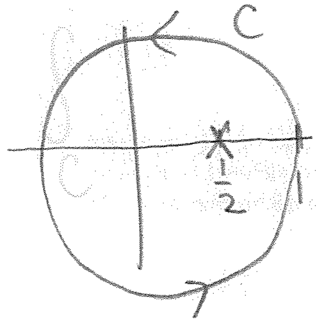
$$= 2\pi i\left(\frac{(-2)^3+5(-2)^2-2(-2)+1}{(-2)-2}\right)$$

$$= 2\pi i\left(\frac{-8+20+4+1}{-4}\right)$$

$$= 2\pi i\left(\frac{17}{-4}\right)$$

$$= -\frac{17\pi i}{2}$$

g)



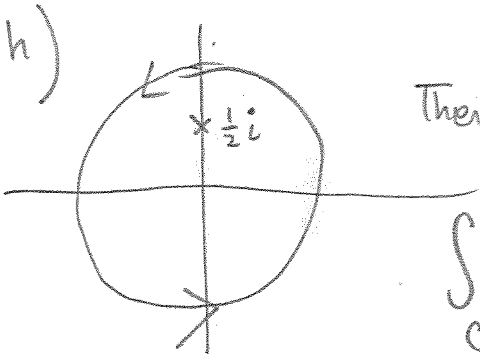
Therefore $z=\frac{1}{2}$ is a blow up point interior to the curve C.

Compute

$$\int_C \frac{z\exp(e^z)}{z-\frac{1}{2}}\,dz \overset{\text{CIF}}{=} \cancel{2}\pi i\left(\frac{1}{\cancel{2}}\exp(e^{\frac{1}{2}})\right)$$

$$= \pi i e^{\sqrt{e}}$$

h)



Therefore $z=\frac{1}{2}i$ is interior to C. Compute

$$\int_C \frac{z\exp(e^z)}{z-\frac{1}{2}i}\,dz \overset{\text{CIF}}{=} \cancel{2}\pi i\left(\frac{1}{\cancel{2}}i\exp(e^{\frac{1}{2}i})\right)$$

$$= -\pi\exp\left(\cos(\tfrac{1}{2})+i\sin(\tfrac{1}{2})\right)$$

$$= -\pi e^{\cos(\frac{1}{2})}\left[\cos(\sin(\tfrac{1}{2}))+i\sin(\sin(\tfrac{1}{2}))\right]$$