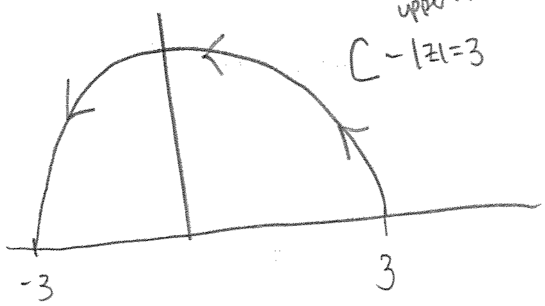
HW8 MATH 1199 Fall 2019

(1)



$C \sim |z|=3$ upper half of

Bound

$$\left|\int_C \frac{z+1}{z^2-5}\,dz\right|$$

Soln: By Δ-ineq, for z on C,

$$|z+1| \le |z|+1 = 3+1 = 4$$

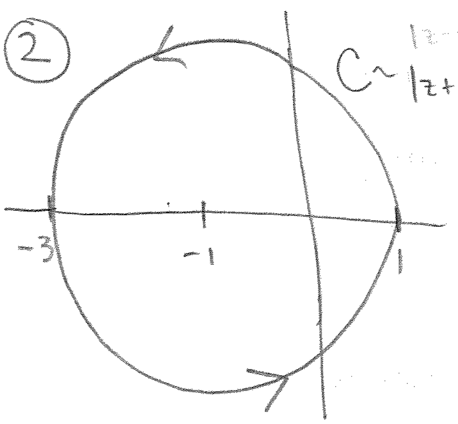
and

$$|z^2-5| \ge \big||z|^2-5\big| = |3^2-5| = 4 \implies \frac{1}{|z^2-5|} \le \frac{1}{4}$$

Therefore, $\left|\frac{z+1}{z^2-5}\right| \le \frac{4}{4} = 1 \leftarrow M$ and length of C is $3\pi \leftarrow L$, so by ML-ineq,

$$\left|\int_C \frac{z+1}{z^2-5}\,dz\right| \le ML = 3\pi$$

(2)



$|z-(-1)|=2$

$C \sim |z+1|=2$ Bound $\left|\int_C \frac{3z-1}{(z+1)^2+1}\,dz\right|$.

Soln: By Δ-ineq, for z on C,

need $|z+1|$ to use condition that defines C

$$|3z-1| = |3(z+1)-4| \le 3|z+1|+4 = 3(2)+4 = 10$$

and

$$|(z+1)^2+1| \ge \big||z+1|^2+1\big| = |4+1| = 5. \implies \left|\frac{1}{(z+1)^2+1}\right| \le \frac{1}{5}$$

Therefore, $\left|\frac{3z-1}{(z+1)^2+1}\right| \le \frac{10}{5} = 2 \leftarrow M$ and the length of C is $2\pi(2) = 4\pi \leftarrow L$

Therefore, by ML-inequality,

$$\left|\int_C \frac{3z-1}{(z+1)^2+1}\,dz\right| \le ML = 8\pi$$