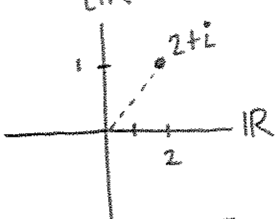
HW1 MATH 1199 FALL 2019

1) a) $(2+i)(1-i) = 2-2i+i-i^2$
$= 2-i+1$
$= 3-i$

b) $\dfrac{2}{1+i} = \dfrac{2}{1+i}\left(\dfrac{1-i}{1-i}\right) = \dfrac{2-2i}{1-i^2} = \dfrac{2-2i}{2} = 1-i$

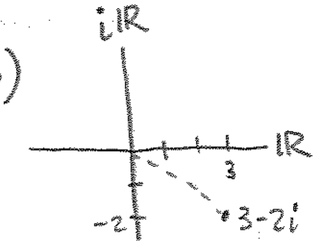
c) $\dfrac{1+i}{1-i} + \dfrac{2}{i} = \dfrac{(i+i^2)+(2-2i)}{i-i^2} = \dfrac{1-i}{1+i} = \left(\dfrac{1-i}{1+i}\right)\left(\dfrac{1-i}{1-i}\right)$
$= \dfrac{1-2i+i^2}{1-i^2}$
$= \dfrac{-2i}{2}$
$= -i$

2) a)



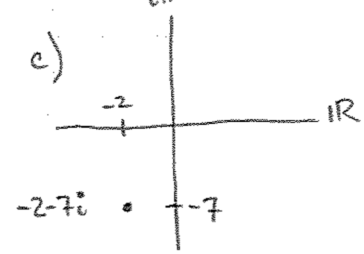
$|2+i| = \sqrt{2^2+1^2} = \sqrt{5}$

b)



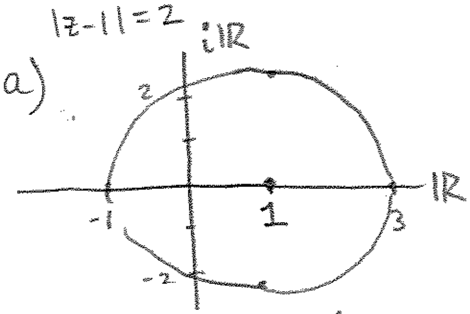
$|3-2i| = \sqrt{3^2+(-2)^2} = \sqrt{13}$

c)



$|-2-7i| = \sqrt{(-2)^2+(-7)^2}$
$= \sqrt{53}$

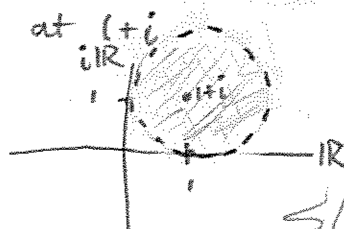
3) a) $|z-1| = 2$



circle centered at 1 of radius 2

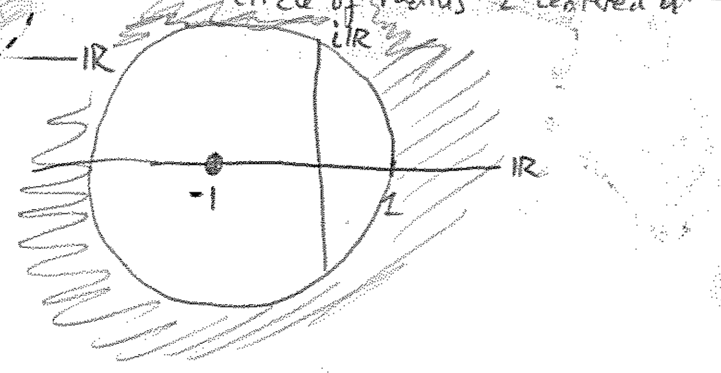
b) $|z-(1+i)| < 1$

interior of circle of radius 1 centered at $1+i$



c) $|z+1| \geq 2$
$\Downarrow$
$|z-(-1)| \geq 2$

exterior to (including boundary) circle of radius 2 centered at -1



4 a) $\overline{\overline{z} - 2i} = \overline{\overline{z}} + \overline{(-2i)} = z + 2i$

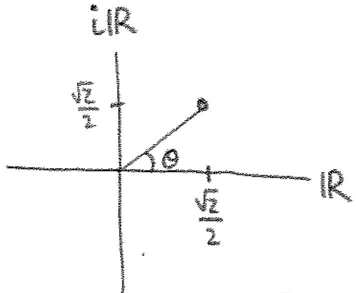
b) if $z = a+bi$, then

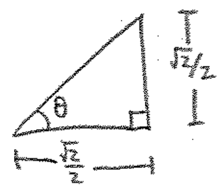
$\overline{iz} = \overline{i(a+bi)} = \overline{ai-b} = -ai-b$, equal!!

while

$-i\overline{z} = -i\overline{(a+bi)} = -i(a-bi) = -ai+bi^2 = -ai-b$

5 a)





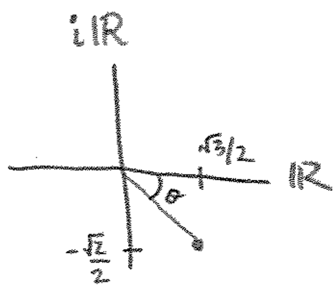
$z = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ is in QI, so

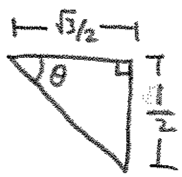
$$\arg(z) = \arctan\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right) + 2n\pi$$
$$= \frac{\pi}{4} + 2n\pi$$

So we take $n=0$ to get

$$-\pi < \mathrm{Arg}(z) = \frac{\pi}{4} \leq \pi$$

b)





$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$ in QIV $\Longrightarrow$ $\arg(z) = \arctan\left(\frac{-1/2}{\sqrt{3}/2}\right) + 2n\pi$

$$= \arctan\left(-\frac{1}{\sqrt{3}}\right) + 2n\pi$$
$$= -\frac{\pi}{6} + 2n\pi$$

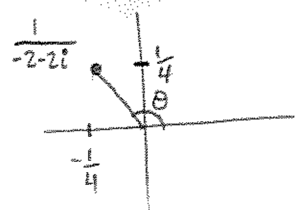
So take $n=0$ to get

$$-\pi < \mathrm{Arg}(z) = -\frac{\pi}{6} \leq \pi$$

c) Since $\frac{1}{-2-2i}$ is not written in form $a+bi$, we first put it into
that form:

$$\frac{1}{-2-2i} = \left(\frac{1}{-2-2i}\right)\left(\frac{-2+2i}{-2+2i}\right) = \frac{-2+2i}{4-4i^2} = \frac{-2+2i}{8} = -\frac{1}{4}+\frac{1}{4}i$$

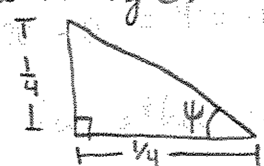
Therefore $\frac{1}{-2-2i}$ lies in QII :



Two ways to proceed:

Way 1

Draw triangle:



, where $\psi + \theta = \pi$ in this situation.

Therefore

$$\psi = \arctan\left(\frac{1/4}{1/4}\right) = \frac{\pi}{4}$$

and

$$\frac{\pi}{4} = \psi = \pi - \theta$$

$$\Rightarrow \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Way 2

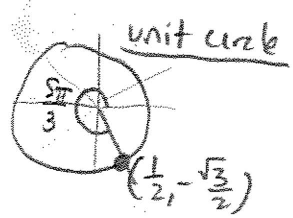
Use formula

$$\arg\left(\frac{1}{-2-2i}\right) = \pi + \arctan\left(\frac{1/4}{-1/4}\right)$$

$$= \pi + \arctan(-1)$$

$$= \pi + \left(-\frac{\pi}{4}\right)$$

$$= \frac{3\pi}{4}$$



#6)
$$e^{\frac{5\pi}{3}i} = \cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)$$

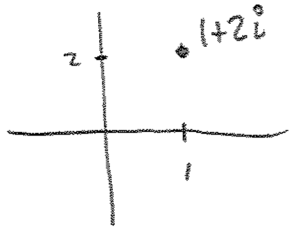
$$= \frac{1}{2} - \frac{\sqrt{3}}{2}i,$$

so

$$\left|e^{\frac{5\pi}{3}i}\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1$$

#7

a)



$r = |1+2i| = \sqrt{1^2+2^2} = \sqrt{5}$

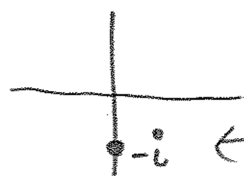
no "nice" answer here
so keep "arctan"

$\mathrm{Arg}(1+2i) = \arctan\left(\frac{2}{1}\right) \approx 1.107$

Therefore

$$1+2i = \sqrt{5}\, e^{\arctan(2) i}$$

b) $\frac{1}{i} = \left(\frac{1}{i}\right)\left(\frac{-i}{-i}\right) = \frac{-i}{\underbrace{-i^2}_{=1}} = -i$

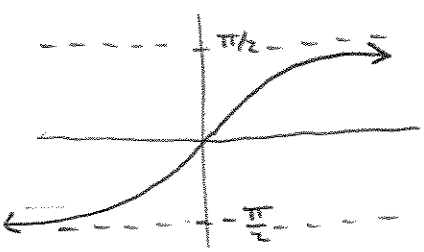


Alternatively, think: what angle in range $-\pi < \theta \leq \pi$ is $-i$? Of course: $-\frac{\pi}{2}$, NOT $\frac{3\pi}{2}$

Here, $\left|\frac{1}{i}\right| = |-i| = \sqrt{0^2+1^2} = 1$

and

$\mathrm{Arg}\left(\frac{1}{i}\right) = \mathrm{Arg}(0+(-1)i) = \cancel{\arctan\left(\frac{-1}{0}\right)}$ ~~"uh oh!!"~~



$= \lim_{x \to 0^+} \arctan\left(\frac{-1}{x}\right)$

$= \lim_{x \to -\infty} \arctan(x)$

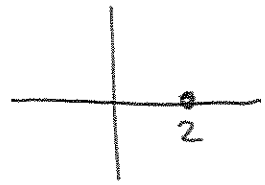
$= -\frac{\pi}{2}$

Therefore,

$$\frac{1}{i} = e^{-\frac{\pi}{2} i}$$

(c) $(1-i)(1+i) = 1-i^2 = 1+1 = 2$

$= 2+0i$



So,

$$|(1-i)(1+i)| = |2+0i| = \sqrt{2^2+0^2} = 2$$

and

$$\operatorname{Arg}\big((1-i)(1+i)\big) = \operatorname{Arg}(2) = \arctan\left(\frac{0}{2}\right) = \arctan(0) = 0$$

Therefore,

$$(1-i)(1+i) = 2e^{0i} = 2$$

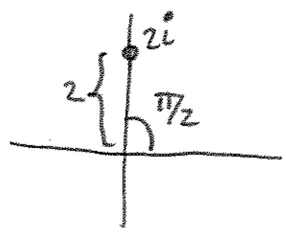
#8)

a) $z = 2e^{\frac{\pi}{2}i}$

$= 2\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right]$

$= 2[0+i]$

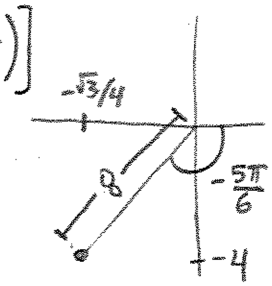
$= 2i$



b) $z = 8e^{-\frac{5\pi}{6}i} = 8\left[\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right]$

$= 8\left[-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right]$

$= -4\sqrt{3} - 4i$



c) $z = 22e^{14\pi i} = 22\left[\cos(14\pi) + i\sin(14\pi)\right]$

$= 22[1 + i(0)]$

$= 22$