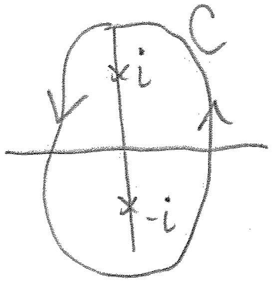
HW 11 MATH 1199 Fall 2019 ①

① ⓐ

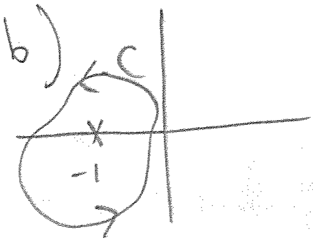


both order 1 poles

$$\int_C \frac{e^z\cos(z)}{z^2+1}\,dz = 2\pi i \operatorname*{Res}_{z=i} \frac{\frac{e^z\cos(z)}{z+i}}{z-i} + 2\pi i \operatorname*{Res}_{z=-i} \frac{\frac{e^z\cos(z)}{z-i}}{z+i}$$

$$= 2\pi i\left(\frac{e^{i}\cos(i)}{2i} + \frac{e^{-i}\cos(-i)}{-2i}\right)$$

b)



order 2 pole

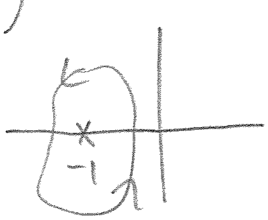
$$\int_C \frac{z^2\sin(z)}{(z+1)^2}\,dz = 2\pi i \operatorname*{Res}_{z=-1} \frac{\overbrace{z^2\sin(z)}^{\phi(z)}}{(z+1)^2}$$

$\phi'(z) = 2z\sin(z) + z^2\cos(z)$

$$= 2\pi i\,\frac{\phi'(-1)}{0!}$$

$$= 2\pi\left[-2\sin(-1)+\cos(-1)\right]$$

c) $z^2+2z+1 = (z+1)^2$



order 2 pole

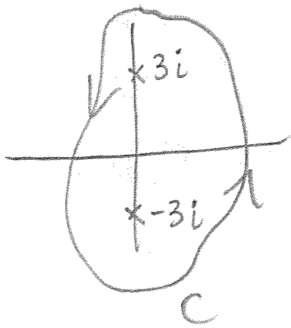
$$\int_C \frac{\cos(z)\sin(z)}{z^2+2z+1}\,dz = 2\pi i \operatorname*{Res}_{z=-1} \frac{\overbrace{\cos(z)\sin(z)}^{\phi(z)}}{(z+1)^2}$$

$$= 2\pi i\,\frac{\phi'(-1)}{0!}$$

$$= 2\pi i\left[\cos^2(-1)-\sin^2(-1)\right]$$

$\phi'(z) = \cos^2(z) - \sin^2(z)$

2) a) $z^2+9=0 \rightarrow z=\pm 3i$



poles of order 1

$$f(t)=\frac{1}{2\pi i}\int_C \frac{e^{zt}}{z^2+9}\,dz$$

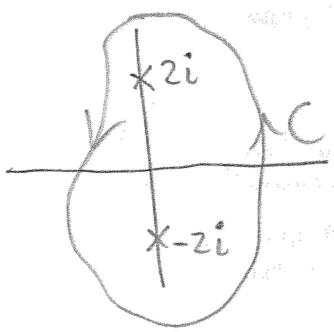
$$=\frac{1}{2\pi i}\left[2\pi i \operatorname*{Res}_{z=3i} \frac{\left(\frac{e^{zt}}{z+3i}\right)}{z-3i} + 2\pi i \operatorname*{Res}_{z=-3i} \frac{\left(\frac{e^{zt}}{z-3i}\right)}{z+3i}\right]$$

$$=\frac{e^{3it}}{6i}+\frac{e^{-3it}}{-6i}$$

$$=\frac{1}{3}\left[\frac{e^{3ti}-e^{-3ti}}{2i}\right]$$

$$=\frac{1}{3}\sin(3t)$$

b) $z^2+4=0 \rightarrow z=\pm 2i$



$$f(t)=\frac{1}{2\pi i}\int_C \frac{ze^{zt}}{z^2+4}\,dz$$

$$=\frac{1}{2\pi i}\left[2\pi i \operatorname*{Res}_{z=2i} \frac{\frac{ze^{zt}}{z+2i}}{z-2i} + 2\pi i \operatorname*{Res}_{z=-2i} \frac{\frac{ze^{zt}}{z-2i}}{z+2i}\right]$$

$$=\frac{2ie^{2it}}{4i}+\frac{(-2i)e^{-2it}}{-4i}$$

$$=\frac{e^{2ti}+e^{-2ti}}{2}$$

$$=\cos(2t)$$

3

$$\phi_1'(z) = \frac{(z+b)^2 te^{zt} - e^{zt}(2(z+b))}{(z+b)^4} = \frac{e^{zt}\left[t(z+b)-2\right]}{(z+b)^3}$$

c) poles of order 2 at $z=-a$ and $z=-b$

$$\phi_2'(z) = \frac{(z+a)^2 te^{zt} - e^{zt}(2(z+a))}{(z+a)^4} = \frac{e^{zt}\left[t(z+a)-2\right]}{(z+a)^3}$$

$$f(t) = \frac{1}{2\pi i}\int_C \frac{e^{zt}}{(z+a)^2(z+b)^2}\,dz$$

$$= \frac{1}{2\pi i}\left[2\pi i \operatorname*{Res}_{z=-a} \frac{\overbrace{\frac{e^{zt}}{(z+b)^2}}^{\phi_1(z)}}{(z+a)^2} + 2\pi i \operatorname*{Res}_{z=-b} \frac{\overbrace{\frac{e^{zt}}{(z+a)^2}}^{\phi_2(z)}}{(z+b)^2}\right]$$

$$= \frac{\phi_1'(-a)}{0!} + \frac{\phi_2'(-b)}{0!}$$

$$= \frac{e^{-at}\left[t(b-a)-2\right]}{(b-a)^3} + \frac{e^{-bt}\left[t(a-b)-2\right]}{(a-b)^3}$$