Homework 9 — MATH 4590 Spring 2018

1. Use induction to prove that

$$1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

2. Define

$$F(x) = \begin{cases} 0, & x < 0 \\ (n-1)x - \dfrac{(n-1)n}{2}, & x \in [n-1, n), n \in \{1, 2, 3, \ldots\} \end{cases}$$

   a.) Sketch this function on $[0, 5]$. Is $F$ continuous?

   b.) Find $F'(x)$ at places which have a derivative. Add a sketch for $F'$ to your sketch in part a.).

   c.) Use the above part to evaluate $\int_a^b \lfloor x \rfloor \mathrm{d}x$, where $\lfloor x \rfloor$ denotes the floor function (i.e. $\lfloor x \rfloor$ is the greatest integer $\leq x$)

3. Prove that if $Z_1$ and $Z_2$ are zero sets, then $Z_1 \cup Z_2$ is a zero set.

4. Show that any two antiderivatives of a function $f$ differ by a constant. (*hint: use the "antiderivative theorem"*)

5. If $f_n \rightrightarrows f$ and $g_n \rightrightarrows g$ on $A$, prove that $f_n + g_n \rightrightarrows f + g$ on $A$.