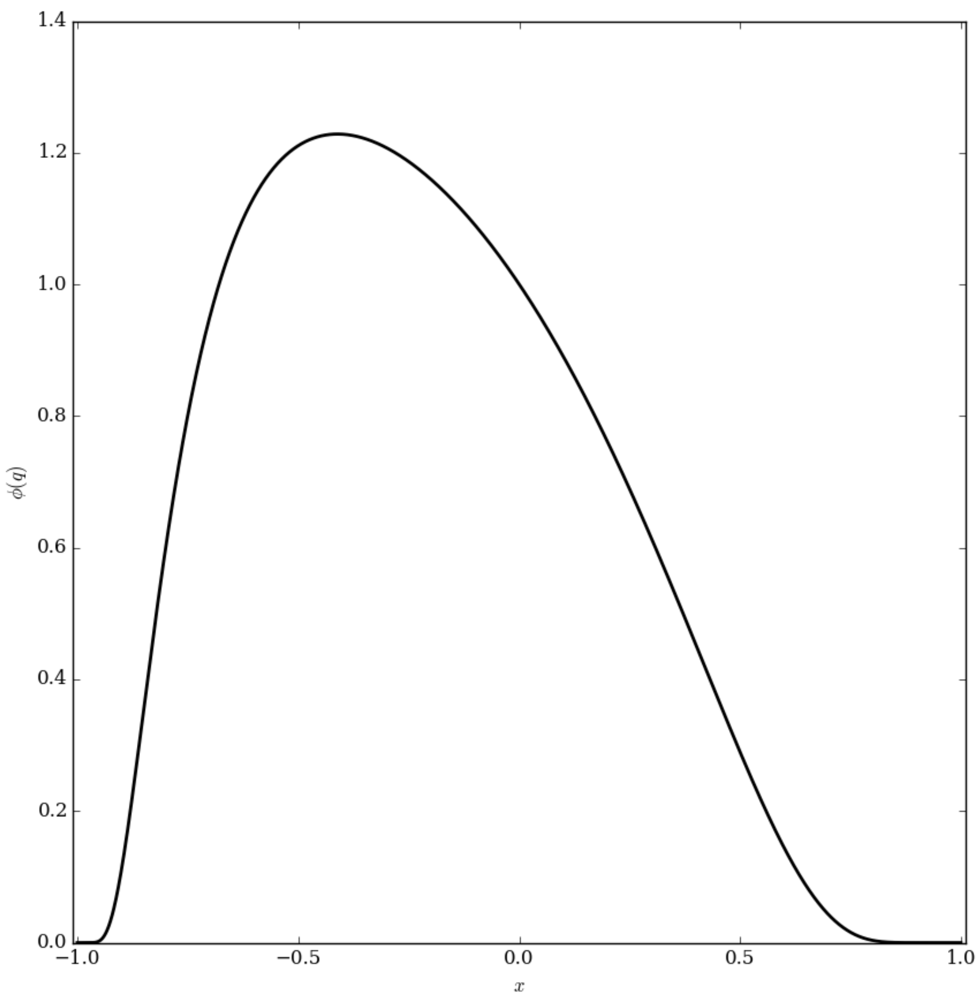
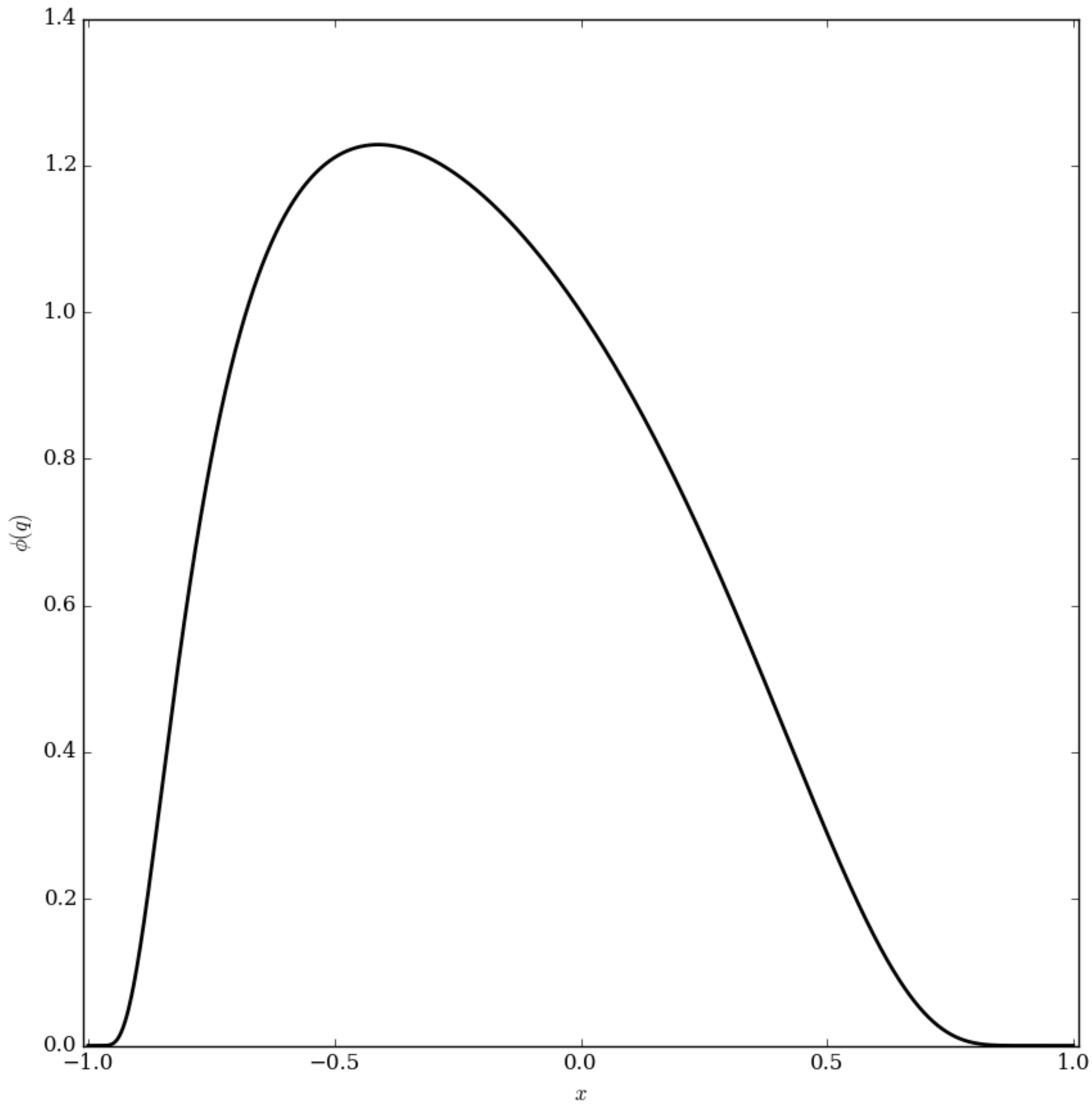
Homework 8 — MATH 4590 (Spring 2018)

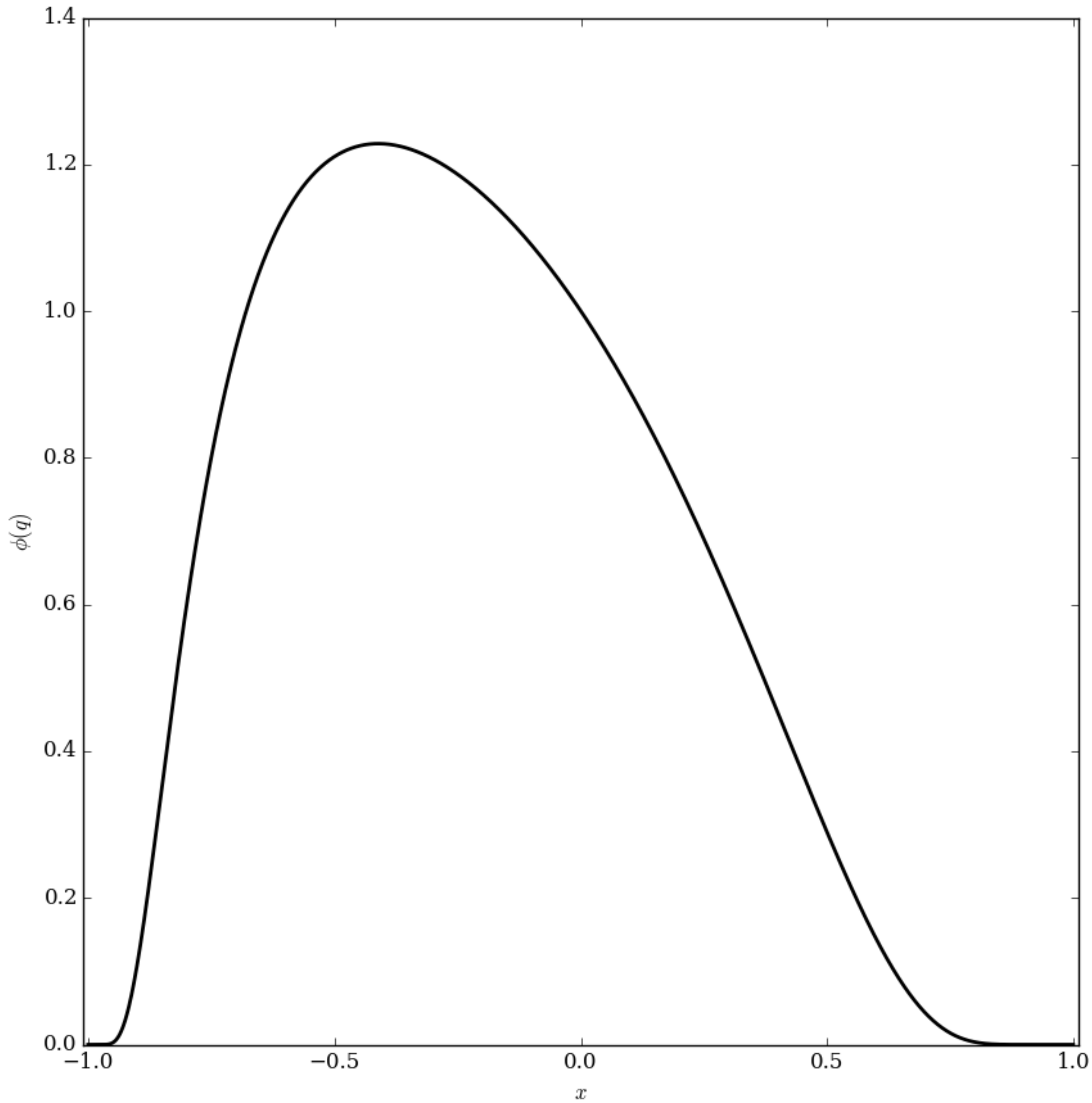
1. The Euler phi function $\phi\colon (-1,1) \to \mathbb{R}$ is defined by $\phi(x) = \prod_{k=1}^{\infty} 1 - x^k$ (this is an "infinite product"). Estimate the value of the integral $\int_{-0.5}^{0.5} \phi(x)\mathrm{d}x$ using a Riemann sum with a partition pair P, T where P contains 5 total points (draw and label everything involved). A plot of the function is shown:



2. Estimate the value of $\int_{-0.5}^{0.5} \phi(x)\mathrm{d}x$ using the Darboux approach with a partition P containing 5 points. Compute both U and L to estimate (draw and label everything involved). The picture of the function appears again for your convenience:



3. Repeat problem 2 but now use a partition containing 8 points.



4. Prove that if the Riemann integral exists, then it is unique.
hint: suppose it is not unique, i.e. there are two different numbers I_1 *and* I_2 *that the integral takes; pick partitions with appropriate meshes according to the definition of Riemann integrability and show that* $\forall \epsilon > 0$, $|I_1 - I_2| < \epsilon$*...the triangle inequality will be useful!*
5. Suppose that f is continuous on $[a, b]$ and that $f(x) \geq 0$ for all $x \in [a, b]$. Also suppose that $\int_a^b f(x)\mathrm{d}x = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.