Homework 6 — MATH 4590 Spring 2018

1.) Chapter 3, # 6 from the text: If $f\colon (a,b) \to \mathbb{R}$ assumes a maximum or minimum at some $\theta \in (a,b)$, prove that $f'(\theta) = 0$.

2.) Prove using induction and the rules of differentiation (or directly, if you want) that the derivative of x^n is nx^{n-1}.

3.) The notion of an "absolute derivative" in metric spaces was recently defined: let (M, d_1) and (N, d_2) be metric spaces and let $f\colon M \to N$ be a function. The absolute derivative of f, $f^{|'|}\colon M \to [0,\infty)$, is defined by

$$f^{|'|}(x) = \lim_{t \to x} \frac{d_2(f(x), f(t))}{d_1(x,t)}.$$

Fix a point $y \in N$ and let f be the constant function $f(x) = y$. Prove that $f^{|'|}(x) = 0$.

4.) Prove that if $f\colon \mathbb{R} \to \mathbb{R}$ (taking the usual metric in each case) is differentiable, then $f^{|'|}(x) = |f'(x)|$. $\Big($*hint: you may find the fact that, for a function* g, $\lim_{t \to x} |g(x)| = \left|\lim_{t \to x} g(x)\right|$ *useful*$\Big)$