Homework 5 – MATH 4590 Spring 2018

1. Let (M, d) be a metric space and let $U \subset M$. Prove that U is open if and only if none of its points are a limit of its complement.

2. Prove that the union of finitely many closed sets is a closed set.

3. Consider $(M, d) = (\mathbb{R}^2, d)$, where d is the Euclidean metric. We define the distance between two nonempty sets $A, B \subset \mathbb{R}^2$ by the following:

$$\operatorname{dist}(A, B) = \inf\{d(a, b) \colon a \in A, b \in B\}.$$

Give an example of two nonempty *disjoint* sets $A, B \subset \mathbb{R}^2$ such that $\operatorname{dist}(A, B) = 0$.