Homework 4 — MATH 4590 Spring 2018

Let (M, d) be a metric space and let $S \subset M$. A point $p \in M$ is called a limit of S provided that there exists a sequence (p_n) in S such that $p_n \to p$. We say that S is closed if it contains all of its limits. We say that S is open if $\forall p \in S \exists r > 0$ such that if $d(p, q) < r$, then $q \in S$ (in other words: you can make a small enough "ball" around p that all points in the ball are also in S).

1. Let $(M, d) = (\mathbb{R}^2, d)$, where d is the Euclidean metric

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

   Is the set $\mathbb{R} = \{(x, 0) \colon x \in \mathbb{R}\} \subset \mathbb{R}^2\}$ a closed, an open, both closed and open, or neither a closed nor an open subset of $\mathbb{R}^2$? Why or why not? (*hint: the set of points a certain distance away from a point in* $\mathbb{R}^2$*, in this metric, looks like a little disk (not including its boundary)*)

2. Let (M, d) be a metric space and let $S = \emptyset$ (i.e. the empty set). Is $\emptyset$ an open set, a closed set, both open and closed, or neither open nor closed? Why or why not?

3. Let (M, d) be a metric space and let $x \in M$ be some arbitrary point. Prove that $S = \{x\}$ is a closed set.