Homework 3 — MATH 4590 Spring 2018

1. Prove that if $M = \mathbb{R}$ and $d(x, y) = |x - y|$, then (M, d) is a metric space.
2. Prove the following "ϵ-principle": if for all $\epsilon > 0$ it holds that $|x - y| < \epsilon$, then $x = y$.
3. Prove that if $M = \mathbb{R}^2$ and $d((x_1, y_1), (x_2, y_2)) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$, then (M, d) is a metric space. Draw the "unit circle" in this metric space, i.e. the set $\{x \in \mathbb{R}^2 \colon d(x, 0) = 1\}$.
4. Let (M, d) be any metric space and let (x_n) and (y_n) be convergent sequences in M with $x_n \to x$ and $y_n \to y$. Prove that the sequence $(d(x_n, y_n))$ is a convergent sequence in $\mathbb{R}$ with $d(x_n, y_n) \to d(x, y)$. (*note: you don't have a formula for the metric d in M nor do you know much about M – it could be a lot of things – but you should be thinking of the metric in $\mathbb{R}$ as $d_{\mathbb{R}}(x, y) = |x - y|$*)
5. Consider the metric space $(\mathbb{R}, d)$ where $d(x, y) = |x - y|$. Prove that the function $f \colon \mathbb{R} \to \mathbb{R}$ with $f(x) = x$ is continuous.