HW 11 (MATH 4590)

① a) Computing $\int_{-1}^{1} (1-x^2)^n \, dx$ for...

$n=0$: $= 2$

$n=1$: $= 4/3$

$n=2$: $= 16/15$

$n=3$: $= 32/35$

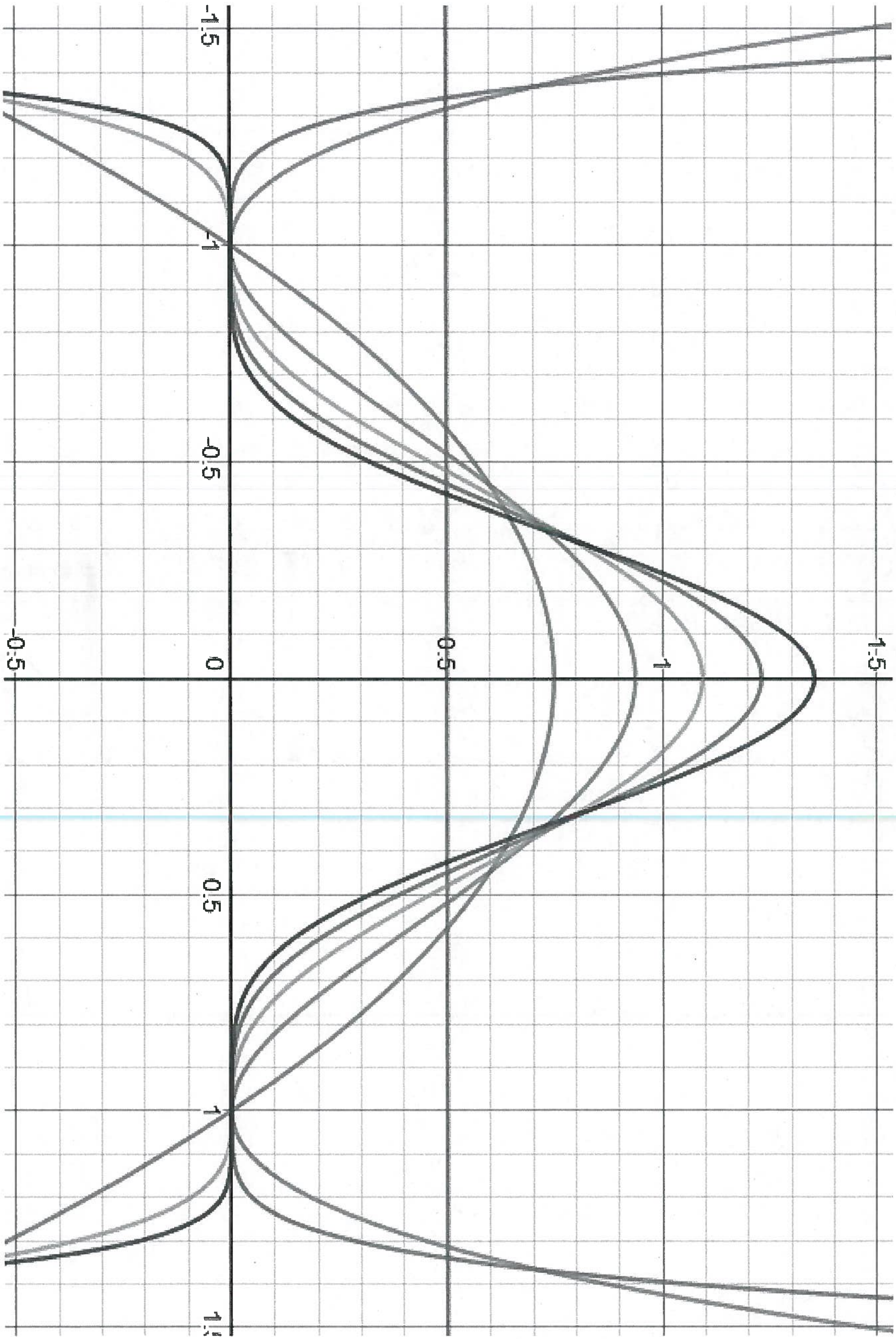
$n=4$: $256/315$

$n=5$: $512/693$

b)

n	$c_n(1-x^2)^n$
0	$\frac{1}{2}$
1	$(3/4)(1-x^2)$
2	$(15/16)(1-x^2)^2$
3	$(35/32)(1-x^2)^3$
4	$(315/256)(1-x^2)^4$
5	$(693/512)(1-x^2)^5$

c)



2) (a) $g'(x) = n(1-x^2)^{n-1}(-2x) + 2nx$

(b) positive (plot them for various n and observe)

(c) it means that g is increasing

(d) $g(0) = (1-0^2)^n - 1 + 0 = 1 - 1 = 0$

(e) $g(x) > 0$ for $x \in [0,1]$

(f) From "$g(x) > 0$" we write

$$(1-x^2)^n - 1 + nx^2 \geq 0 \quad \text{for } x \in [0,1]$$

$\Downarrow$ ALGEBRA

$$(1-x^2)^n \geq 1 - nx^2 \quad \text{for } x \in [0,1]$$

3) (a)

$$\lim_{n\to\infty} Q_n(x) = \lim_{n\to\infty} \sqrt{n}(1-x^2)^n$$

$$= \lim_{n\to\infty} \frac{\sqrt{n} \nearrow \infty}{(1-x^2)^{-n} \searrow \infty}$$

$$\overset{\text{L.H.}}{=} \lim_{n\to\infty} \frac{1/2\sqrt{n}}{-\ln(1-x^2)(1-x^2)^{-n}}$$

$$= \frac{-1}{\ln(1-x^2)} \lim_{n\to\infty} \frac{(1-x^2)^n \to 0}{2\sqrt{n} \to \infty}$$ but is in denominator

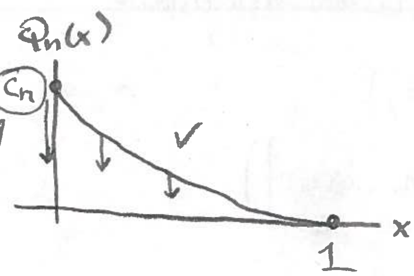
$$= 0$$

$$\frac{d}{dn}(1-x^2)^{-n} = \frac{d}{dn} e^{-n\ln(1-x^2)}$$

$$= -\ln(1-x^2) e^{-n(1-x^2)}$$

$$= -\ln(1-x^2)(1-x^2)^{-n}$$

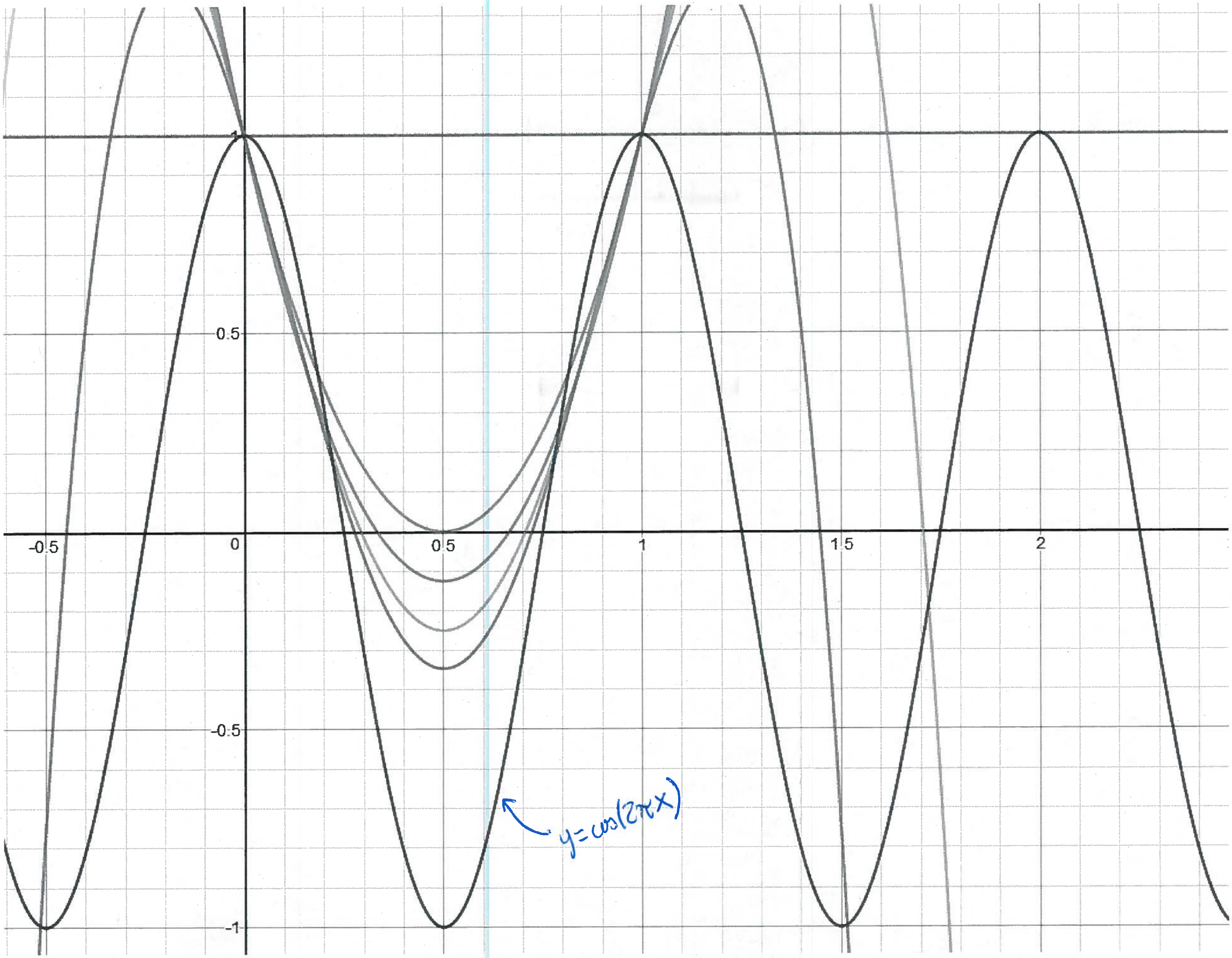
(b)



turns out $C_n \to 0$ as $n \to \infty$

Yes $Q_n \rightrightarrows 0$ because $Q_n(x) \xrightarrow{n\to\infty} 0$ for $x \in [0,1]$ with no problem. At $x_0 = 0$; $Q_n(0) = C_n$ and $C_n \to 0$, so we do have uniform convergence

(c) uniform on $[0,1] \longrightarrow$ uniform on $[\delta, 1]$



-0.5
0
0.5
1
1.5
2
0.5
-0.5
-1
y=cos(2πx)