

Homework 10 — MATH 4590 Spring 2018

1. Let $f_n: [0, 2] \rightarrow \mathbb{R}$ be defined by $f_n(x) = \frac{x^n}{1+x^n}$. Take the limit as $n \rightarrow \infty$ and explain why it shows that convergence is not uniform.
2. Find a sequence of a functions $f_n: [0, 1] \rightarrow \mathbb{R}$ which is discontinuous everywhere for every n but converges to a function that is continuous everywhere. (*hint: think about a function we have seen in the past that is discontinuous everywhere*)
3. Let $g_n: [0, \infty) \rightarrow \mathbb{R}$ be defined by $g_n(x) = \frac{e^{-nx}}{n}$. Examine the relationship between the limit function, the derivative of the limit function, g'_n , and the limit of g'_n .
4. Let $g_n: [0, 1] \rightarrow \mathbb{R}$ be defined by $g_n(x) = nx(1-x)^n$. Discuss the convergence of g_n and the convergence of $\int_0^1 g_n(x)dx$.

5. Let

$$f_n(x) = \begin{cases} 1, & 0 < x < \frac{1}{n} \\ 0, & x = 0 \text{ or } \frac{1}{n} \leq x \leq 1. \end{cases}$$

What is the limit function? Is the convergence uniform?