Quiz 4 MATH 3503 Fall 2018

If $\vec{a}(t) = \langle 1, 2, t \rangle$ and $\vec{v}(1) = \langle 0, 1, 0 \rangle$ and $\vec{r}(1) = \langle -1, 0, 0 \rangle$, find $\vec{r}(t)$.

Soln: First compute

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle t, 2t, \tfrac{t^2}{2} \rangle + \vec{c}$$

So,

$$\underbrace{\langle 0, 1, 0 \rangle}_{\text{given}} = \vec{v}(1) = \underbrace{\langle 1, 2, \tfrac{1}{2} \rangle}_{\text{computed}} + \vec{c}$$

Therefore

$$\vec{c} = \langle 0, 1, 0 \rangle - \langle 1, 2, \tfrac{1}{2} \rangle = \langle -1, -1, -\tfrac{1}{2} \rangle$$

Thus

$$\vec{v}(t) = \langle t, 2t, \tfrac{t^2}{2} \rangle + \langle -1, -1, -\tfrac{1}{2} \rangle = \langle t-1, 2t-1, \tfrac{t^2}{2} - \tfrac{1}{2} \rangle$$

Now compute

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \tfrac{t^2}{2} - t, t^2 - t, \tfrac{t^3}{6} - \tfrac{1}{2}t \rangle + \vec{d}$$

So,

$$\underbrace{\langle -1, 0, 0 \rangle}_{\text{given}} = \vec{r}(1) = \underbrace{\langle \tfrac{1}{2} - 1, 1 - 1, \tfrac{1}{6} - \tfrac{1}{2} \rangle}_{\text{computed}} + \vec{d}$$

$$\langle -1, 0, 0 \rangle = \langle -\tfrac{1}{2}, 0, -\tfrac{1}{3} \rangle + \vec{d}$$

Hence,

$$\vec{d} = \langle -1, 0, 0 \rangle - \langle -\tfrac{1}{2}, 0, -\tfrac{1}{3} \rangle$$
$$= \langle -\tfrac{1}{2}, 0, \tfrac{1}{3} \rangle$$

Thus,

$$\vec{r}(t) = \langle \tfrac{t^2}{2} - t, t^2 - t, \tfrac{t^3}{6} - \tfrac{1}{2}t \rangle + \langle \tfrac{3}{2}, 0, \tfrac{1}{3} \rangle$$
$$= \langle \tfrac{t^2}{2} - t - \tfrac{1}{2}, t^2 - t, \tfrac{t^3}{6} - \tfrac{1}{2}t + \tfrac{1}{3} \rangle$$