Problem (#75, pg. 42) Write the equation of the tangent line in Cartesian coordinates for the given parameter t:

$$\begin{cases} x = e^{\sqrt{t}} \\ y = 1 - \ln(t^2) \\ t = 1. \end{cases}$$

Solution: By Theorem 1.1 (pg. 28), we first compute the derivatives

$$x'(t) = e^{\sqrt{t}} \underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\sqrt{t}}_{\text{chain rule}} = \frac{1}{2\sqrt{t}} e^{\sqrt{t}}$$

and

$$y'(t) = 0 - \frac{1}{t^2} \underbrace{\frac{\mathrm{d}}{\mathrm{d}t} t^2}_{\text{chain rule}} = -\frac{2t}{t^2} = -\frac{2}{t}.$$

Now we may compute

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y'(t)}{x'(t)} = \frac{-\frac{2}{t}}{\frac{e^{\sqrt{t}}}{2\sqrt{t}}} = \frac{-2}{t}\frac{2\sqrt{t}}{e^{\sqrt{t}}} = \frac{-4}{\sqrt{t}e^{\sqrt{t}}}.$$

To find the tangent line, we need to find the slope at the point corresponding to parameter $t = 1$:

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=1} = \frac{-4}{\sqrt{1}e^{\sqrt{1}}} = -\frac{4}{e}.$$

To finish the problem, we must use the point-slope form of the line:

$$y - y_0 = \text{slope}(x - x_0),$$

so we have to find y_0 and x_0 – these are just $x(t)$ and $y(t)$ at the parameter $t = 1$:

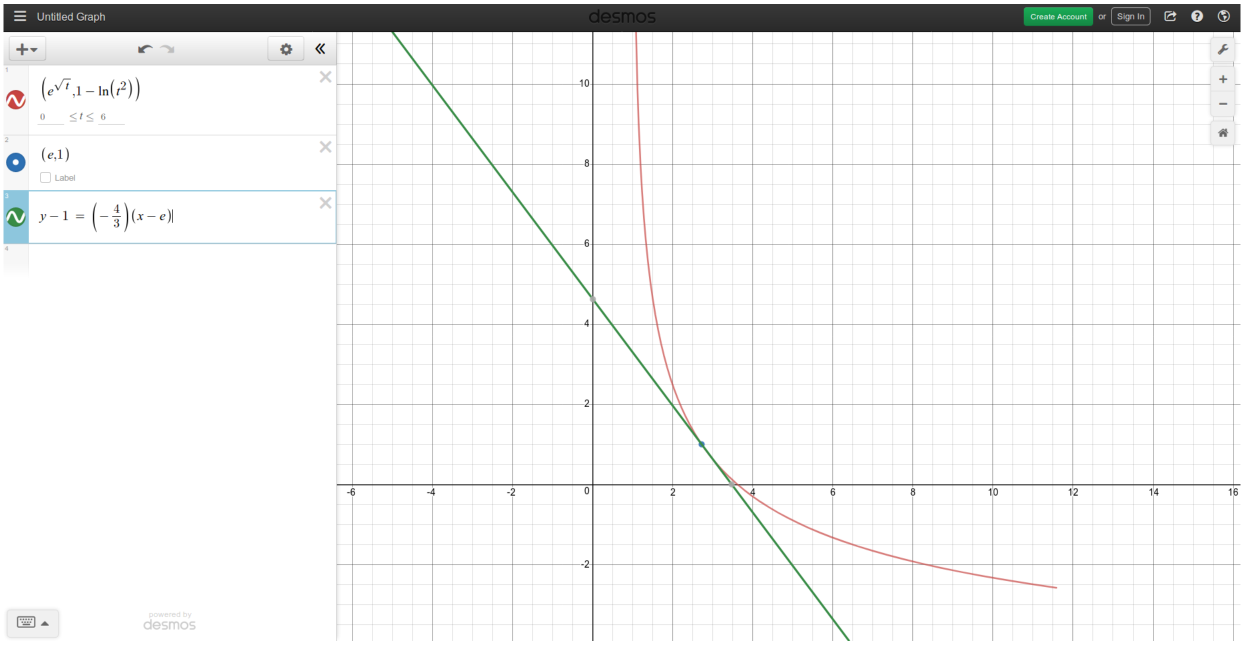
$$x(1) = e^{\sqrt{1}} = e$$

and

$$y(1) = 1 - \ln(1^2) = 1 - \ln(1) = 1 - 0 = 1.$$

Thus we have $x_0 = e$ and $y_0 = 1$. We may now write the equation of the tangent line:

$$y - 1 = \left(-\frac{4}{3}\right)(x - e)$$



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$\left(e^{\sqrt{t}}, 1-\ln\left(t^2\right)\right)$
$0 \le t \le 6$
$(e, 1)$
Label
$y-1=\left(-\frac{4}{3}\right)(x-e)$
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