**Problem** (#102, pg. 43) Find the area enclosed by the ellipse

$$\begin{cases} x = a\cos(\theta) \\ y = b\sin(\theta) \\ 0 \le t \le 2\pi. \end{cases}$$

*Solution*: Using Theorem 1.2, we first compute

$$x'(\theta) = -a\sin(\theta).$$

Now the theorem tells us that the area is given by (note: the $a$ and $b$ in the theorem are the bounds on the parameter $t$ – they are not the same as the $a$ and $b$ in this problem!!)

$$\begin{aligned} \text{Area} &= \int_0^{2\pi} y(\theta)x'(\theta)\mathrm{d}\theta \\ &= \int_0^{2\pi} (-a\sin(\theta))(b\sin(\theta))\mathrm{d}\theta \\ &= -ab\int_0^{2\pi} \underbrace{\sin^2(\theta)}_{=\frac{1-\cos(2\theta)}{2}}\,\mathrm{d}\theta \\ &= -ab\int_0^{2\pi}\frac{1}{2}\mathrm{d}\theta - ab\int_0^{2\pi}\frac{\cos(2\theta)}{2}\mathrm{d}\theta \\ &= -\frac{ab}{2}\theta\Big|_{\theta=0}^{\theta=2\pi} - \frac{ab}{4}\sin(2\theta)\Big|_{\theta=0}^{\theta=2\pi} \\ &= -\frac{ab}{2}(2\pi - 0) - \frac{ab}{4}(\underbrace{\sin(4\pi)}_{=0} - \underbrace{\sin(0)}_{=0}) \\ &\stackrel{?}{=} -ab\pi \end{aligned}$$

Note: again we see that the "area" became negative. The actual answer is $ab\pi$. The sign error is a consequence of the direction of our parametrization of the ellipse! Some texts write Theorem 1.2 with a $|x'(t)|$ inside the integral to accomodate this, but our text does not. In short – if you end up with a negative answer for area, just make it positive to get the correct answer.