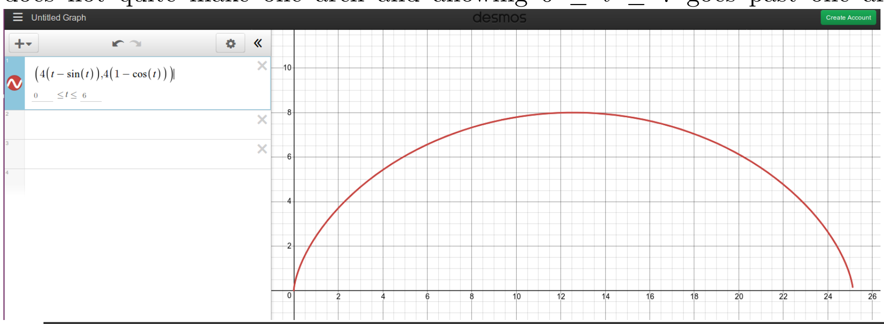
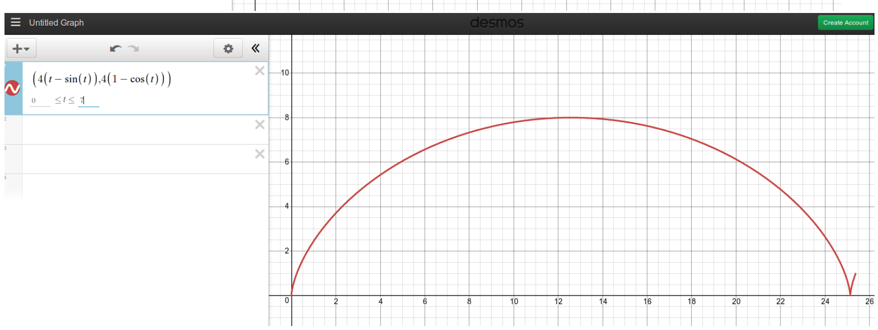
Problem: (#116, pg. 43) Find the arc length of one arch of the cycloid

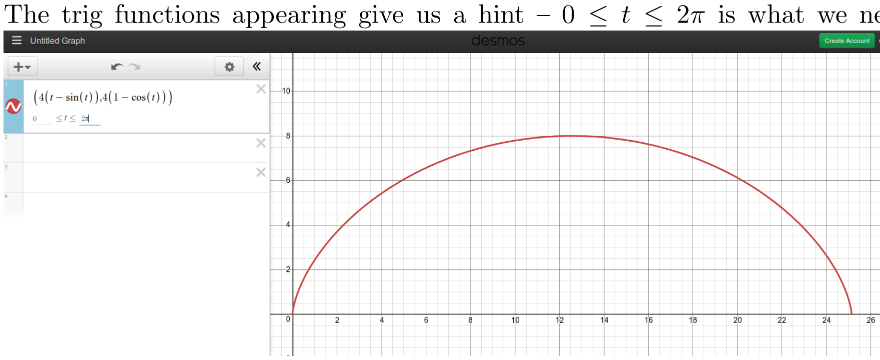
$$\begin{cases} x = 4(t - \sin(t)) \\ y = 4(1 - \cos(t)) \end{cases}$$

Solution: We are not told the bounds on t for this problem – we have to figure them out. Some experimentation in desmos shows that allowing $0 \leq t \leq 6$ does not quite make one arch and allowing $0 \leq t \leq 7$ goes past one arch:





The trig functions appearing give us a hint – $0 \leq t \leq 2\pi$ is what we need!



Now we may use Theorem 1.3 (pg. 37) to compute arc length: first compute

$$x'(t) = 4(1 - \cos(t))$$

and compute

$$y'(t) = 4\sin(t).$$

Now we may use Theorem 1.3 to compute

$$\begin{aligned}
\text{ArcLength} &= \int_0^{2\pi} \sqrt{(4(1-\cos(t))^2 + (4\sin(t))^2}\mathrm{d}t \\
&= \int_0^{2\pi} \sqrt{16 - 32\cos(t) + \underbrace{16\cos^2(t) + 16\sin^2(t)}_{=16(\cos^2(t)+\sin^2(t))=16}}\mathrm{d}t \\
&= \int_0^{2\pi} \sqrt{32 - 32\cos(t)}\mathrm{d}t \\
&= \sqrt{32}\int_0^{2\pi} \sqrt{1-\cos(t)}\mathrm{d}t \\
&\overset{\sqrt{2}\sin(\frac{t}{2})=\sqrt{1-\cos(t)}}{=} \underbrace{\sqrt{64}}_{=8}\int_0^{2\pi} \sin\left(\frac{t}{2}\right)\mathrm{d}t \\
&\overset{u=\frac{t}{2}\to 2\mathrm{d}u=\mathrm{d}t}{=} 16\int_0^{\pi} \sin(u)\mathrm{d}u \\
&= 16(-\cos(u))\Big|_{u=0}^{u=\pi} \\
&= 16(-\cos(\pi) - (-\cos(0))) \\
&= 16(-(-1) - (-1)) \\
&= 16(1+1) \\
&= 32.
\end{aligned}$$