HW8 MATH 3503 Fall 2018

Section 4.5

#216 $w = e^{tv}$, $t = r+s$, $v = rs$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial t}\frac{\partial t}{\partial r} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial r} = (ve^{tv})(1) + (te^{tv})s$$

$$= (rs)e^{(r+s)rs} + s(r+s)e^{(r+s)rs}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial t}\frac{\partial t}{\partial s} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial s}$$

$$= (ve^{tv})(1) + (te^{tv})r$$

$$= rse^{(r+s)rs} + r(r+s)e^{(r+s)rs}$$

#220 $f(x,y) = x+y$, $u = e^x\sin(y)$, $x = t^2$, $y = \pi t$, $x = r\cos(\theta)$, $y = r\sin(\theta)$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= (1)(-r\sin(\theta)) + (1)(r\cos(\theta))$$

$$= -r\sin(\theta) + r\cos(\theta)$$

#224 $f(x,y) = \frac{x}{y}$, $x = e^t$, $y = 2e^t$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= \left(\frac{1}{y}\right)e^t + \left(-\frac{x}{y^2}\right)(2e^t)$$

$$= \frac{e^t}{2e^t} - \frac{e^t}{(2e^t)^2}(2e^t)$$

$$= \frac{1}{2} - 1/2 = 0$$

#244 $z = e^{x^2 y}$, $x = \sqrt{uv}$, $y = \frac{1}{v}$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$= \left(2xy\, e^{x^2 y}\right)\frac{1}{2}\sqrt{\frac{v}{u}} + \left(x^2 e^{x^2 y}\right)(0)$$

$$= \frac{\cancel{2}\cancel{\sqrt{uv}}}{\cancel{v}} e^{\frac{uv}{v}} \cdot \frac{1}{\cancel{2}}\cancel{\sqrt{\frac{v}{u}}} = e^{u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= \left(2xy e^{x^2 y}\right)\sqrt{\frac{u}{v}} + \left(x^2 e^{x^2 y}\right)\left(-\frac{1}{v^2}\right)$$

$$= \left(\frac{2\cancel{\sqrt{uv}}}{v} e^{\frac{uv}{v}}\right)\sqrt{\frac{u}{\cancel{v}}} + \left(uv e^{\frac{uv}{v}}\right)\left(-\frac{1}{v^2}\right)$$

$$= 2\frac{u}{v}e^{u} - \frac{u}{v}e^{u}$$

$$= \frac{u}{v}e^{u}$$

#252 $PV = kT$

$$P = \frac{kT}{V} \xrightarrow{k=1} P = \frac{T}{V}$$

$$\frac{dP}{dt} = \frac{\partial P}{\partial V}\frac{dV}{dt} + \frac{\partial P}{\partial T}\frac{dT}{dt}$$

$$= \left(-\frac{T}{V^2}\right)\frac{dV}{dt} + \left(\frac{1}{V}\right)\frac{dT}{dt}$$

So,

$$\left.\frac{dP}{dt}\right|_{\substack{\frac{dV}{dt}=2,\ \frac{dT}{dt}=\frac{1}{2} \\ V=20,\ T=20}} = \left(\frac{-20}{20^2}\right)(2) + \left(\frac{1}{20}\right)\cdot\frac{1}{2}$$

$$= \frac{-2}{20} + \frac{1}{40}$$

$$= \frac{-4}{40} + \frac{1}{40}$$

$$= -\frac{3}{40}$$