HW7 MATH 3503 Fall 2018

Section 4.2

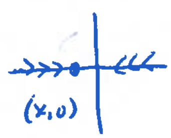
#64) $$\lim_{(x,y)\to(11,13)} \sqrt{\frac{1}{xy}} = \sqrt{\frac{1}{(11)(13)}} = \frac{1}{\sqrt{143}}$$

#66) $$\lim_{(x,y)\to(0,0)} \sin\left(\frac{x^8+x^7}{x-y+10}\right) = \sin\left(\frac{0}{10}\right)$$
$$= \sin(0) = 0$$

#68) $$\lim_{(x,y)\to(0,\frac{\pi}{4})} \frac{\sec(x)+2}{3x-\tan(y)} = \frac{\sec(0)+2}{3(0)-\tan(\pi/4)} = \frac{3}{-1} = -3$$

#86) $$\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$$

a) along x-axis:

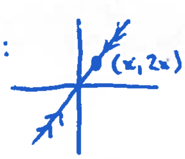


$$\lim_{(x,0)\to(0,0)} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$$

b) along y-axis:

$$\lim_{(0,y)\to(0,0)} \frac{0+y^3}{0+y^2}$$
$$= \lim_{y\to 0} y = 0$$

c) along $y=2x$:



$$\lim_{(x,2x)\to(0,0)} \frac{2x^2+(2x)^3}{x^2+(2x)^2}$$
$$= \lim_{x\to 0} \frac{2x^2+8x^3}{x^2+4x^2} \rightarrow = \frac{0}{0}$$

Recall L'Hôpital's Rule

$$\overset{\text{L.H.}}{=} \lim_{x\to 0} \frac{4x+24x^2}{2x+8x} \rightarrow \frac{0}{0}$$
$$\overset{\text{L.H.}}{=} \lim_{x\to 0} \frac{4+48x}{10}$$
$$= \frac{4}{10} = \frac{2}{5}$$

#87 Because two different paths in #86 yielded different values for the limit, we say that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy+y^3}{x^2+y^2}$ DNE.

#88 $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$

a) along x-axis 

$\lim\limits_{(x,0)\to(0,0)} \dfrac{0}{x^4} = \lim\limits_{x\to 0} 0 = 0$

b) along y-axis 

$\lim\limits_{(0,y)\to(0,0)} \dfrac{0}{y^2} = \lim\limits_{y\to 0} 0 = 0$

c) along $y=x^2$: 

$\lim\limits_{(x,x^2)\to(0,0)} \dfrac{x^4}{2x^4} = \lim\limits_{x\to 0} \dfrac{1}{2} = \dfrac{1}{2}$

#89 Since two different paths yielded different limits in #88, we say that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^4}$ DNE.

Section 4.3

#118 $\dfrac{\partial}{\partial x} \sin(3x)\cos(3y) = 3\cos(3x)\cos(3y)$

#120 $\dfrac{\partial}{\partial x}\left(x^8 e^{3y}\right) = 8x^7e^{3y}$ $\quad\quad$ $\dfrac{\partial}{\partial y}\left(x^8 e^{3y}\right) = 3x^8e^{3y}$

#122 $f_y = \dfrac{\partial}{\partial y}\left(e^{xy}\cos(x)\sin(y)\right) = \cos(x)\dfrac{\partial}{\partial y}\left(e^{xy}\sin(y)\right)$

$= \cos(x)\left[xe^{xy}\sin(y) + e^{xy}\cos(y)\right]$

#124 $\dfrac{\partial}{\partial x} \ln\left(\dfrac{x}{y}\right) = \left(\dfrac{1}{\frac{x}{y}}\right)\left(\dfrac{1}{y}\right) = \dfrac{y}{x}\cdot\dfrac{1}{y} = \dfrac{1}{x}$

$\dfrac{\partial}{\partial y} \ln\left(\dfrac{x}{y}\right) = \left(\dfrac{1}{\frac{x}{y}}\right)\left(-\dfrac{x}{y^2}\right) = \left(\dfrac{y}{x}\right)\left(-\dfrac{x}{y^2}\right) = -\dfrac{1}{y}$

#126 $\dfrac{\partial}{\partial x} \sinh(2x+3y) = \cosh(2x+3y)(2)$

$\dfrac{\partial}{\partial y} \sinh(2x+3y) = \cosh(2x+3y)(3)$

#132) $A = ab\sin(\theta)$

a) $\frac{\partial A}{\partial a} = b\sin(\theta)$ b) $\frac{\partial A}{\partial b} = a\sin(\theta)$ c) $\frac{\partial A}{\partial \theta} = ab\cos(\theta)$

#134) $\frac{\partial}{\partial z}\left(z\sin(xy^2+2z)\right) = \sin(xy^2+2z) + z\cos(xy^2+2z)(2)$

#136) $z = \ln(x-y)$

$$f_{yx} = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\ln(x-y) = \frac{\partial}{\partial x}\left[\left(\frac{1}{x-y}\right)(-1)\right]$$
$$= -\frac{\partial}{\partial x}(x-y)^{-1}$$
$$= +(x-y)^{-2}(1)$$
$$= \frac{1}{(x-y)^2}$$

#138) $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial}{\partial y}\left(e^x\tan(y)\right) = \frac{\partial}{\partial x} e^x\sec^2(y) = e^x\sec^2(y)$

$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{s(x)}{c(x)} = \frac{1}{c^2} = \sec^2 x$

$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\frac{\partial}{\partial x} e^x\tan(y) = \frac{\partial}{\partial y} e^x\tan(y) = e^x\sec^2(y)$

#146) $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[2x^2+2xy+y^2+2x-3\right] = 4x+2y+2 \overset{SET}{=} 0$ (i)

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[2x^2+2xy+y^2+2x-3\right] = 2x+2y \overset{SET}{=} 0$ (ii)

Solve (ii) for x to get $x=-y$. Plug into (i) to get

$4(-y)+2y+2=0 \rightarrow -2y=-2$

$\rightarrow \boxed{y=1} \rightarrow \boxed{x=-1}$

Therefore only place where $\frac{\partial f}{\partial x}=0$ and $\frac{\partial f}{\partial y}=0$ simultaneously is at $(-1,1)$.

#150 Compute

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial}{\partial x}\ln(x^2+y^2) = \frac{\partial}{\partial x}\left(\frac{2x}{x^2+y^2}\right) = \frac{(x^2+y^2)(2) - \overbrace{(2x)(2x)}^{=4x^2}}{(x^2+y^2)^2}$$

$$= \frac{-2x^2+2y^2}{(x^2+y^2)^2}$$

and

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial}{\partial y}\ln(x^2+y^2) = \frac{\partial}{\partial y}\left[\frac{2y}{x^2+y^2}\right] = \frac{(x^2+y^2)(2) - \overbrace{(2y)(2y)}^{4y^2}}{(x^2+y^2)^2}$$

$$= \frac{-2y^2+2x^2}{(x^2+y^2)^2}$$

Therefore, compute

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \left(\frac{\cancel{-2x^2}+\cancel{2y^2}}{(x^2+y^2)^2}\right) + \left(\frac{\cancel{-2y^2}+\cancel{2x^2}}{(x^2+y^2)^2}\right) = 0,$$

as was to be shown.

#156

$$\frac{\partial P}{\partial V} = \frac{\partial}{\partial V}\left[\frac{nRT}{V}\right] = \frac{-nRT}{V^2}$$ ~ rate of change of pressure w.r.t. volume

$$\frac{\partial P}{\partial T} = \frac{\partial}{\partial T}\left[\frac{nRT}{V}\right] = \frac{nR}{V}$$ ~ rate of change of pressure w.r.t. temperature

Section 4.4

#174

$$\frac{\partial z}{\partial x} = \left(e^{7x^2+4y^2}\right)(14x) \quad \text{and} \quad \frac{\partial z}{\partial y} = \left(e^{7x^2+4y^2}\right)(8y)$$

so

$$\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(0,0)} = 0 \quad \text{and} \quad \left.\frac{\partial z}{\partial y}\right|_{(x,y)=(0,0)} = 0$$

Therefore equation of tangent plane is

$$z - 1 = 0(x-0) + 0(y-0)$$

$$\downarrow$$

$$\boxed{z = 1}$$