HW6 MATH 3503 Fall 2018

Section 3.2

#64) $\begin{cases} \vec{a}(t) = \langle -5\cos(t), -5\sin(t) \rangle \\ \vec{v}(0) = \langle 9,2 \rangle, \ \vec{r}(0) = \langle 5,0 \rangle \end{cases}$

Find $\vec{r}(t)$.

Soln: First compute

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left\langle \int -5\cos(t)\,dt, \int -5\sin(t)\,dt \right\rangle$$
$$= \langle -5\sin(t), 5\cos(t) \rangle + \vec{c}$$

Now note that

$$\underbrace{\langle 9,2 \rangle}_{\text{given}} = \vec{v}(0) = \underbrace{\langle 0,5 \rangle + \vec{c}}_{\text{computed}}$$

$$\Rightarrow \vec{c} = \langle 9,2 \rangle - \langle 0,5 \rangle = \langle 9,-3 \rangle$$

Therefore, $\vec{v}(t) = \langle -5\sin(t) + 9, 5\cos(t) - 3 \rangle$

Now compute $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 5\cos(t) + 9t, 5\sin(t) - 3t \rangle + \vec{d}$

Now note that

$$\underbrace{\langle 5,0 \rangle}_{\text{given}} = \vec{r}(0) = \underbrace{\langle 5+0, 0-0 \rangle + \vec{d}}_{\text{computed}}$$

$$\Rightarrow \vec{d} = \langle 5,0 \rangle - \langle 5,0 \rangle = \langle 0,0 \rangle$$

Therefore, $\vec{r}(t) = \langle 5\cos(t) + 9t, 5\sin(t) - 3t \rangle$

#74) $\vec{r}(t) = \langle t, 3t, t^2 \rangle$ and $\vec{u}(t) = \langle 4t, t^2, t^3 \rangle$

Soln: $\frac{d}{dt}\left(\vec{r}(t) \times \vec{u}(t)\right) = \frac{d}{dt}\left(\det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & 3t & t^2 \\ 4t & t^2 & t^3 \end{bmatrix}\right)$

$$= \frac{d}{dt}\langle 3t^4 - t^4, -(t^4 - 4t^3), t^3 - 12t^2 \rangle$$

$$= \frac{d}{dt}\langle 2t^4, 4t^3 - t^4, t^3 - 12t^2 \rangle$$

$$= \langle 8t^3, 12t^2 - 4t^3, 3t^2 - 24t \rangle$$

#87) $\vec{r}(t) = \langle \cos(t), 2\sin(t), 0 \rangle$

Soln: $\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = \langle -\sin(t), 2\cos(t), 0 \rangle$

#88) Soln: Speed at t $= \|\vec{v}(t)\| = \sqrt{(-\sin(t))^2 + 4\cos^2(t) + 0^2} = \sqrt{\sin^2(t) + 4\cos^2(t)}$

Speed at $t = \frac{\pi}{4}$ $= \|\vec{v}(\pi/4)\| = \sqrt{(\sin(\pi/4))^2 + 4(\cos(\pi/4))^2}$

$$= \sqrt{(\sqrt{2}/2)^2 + 4(\sqrt{2}/2)^2}$$

$$= \sqrt{\frac{2}{4} + \frac{8}{4}} = \sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{2}$$

#89) Soln: $\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \langle -\cos(t), -2\sin(t), 0 \rangle$

$$\vec{a}(\pi/4) = \langle -\frac{\sqrt{2}}{2}, -2\left(\frac{\sqrt{2}}{2}\right), 0 \rangle$$

$$= \langle -\frac{\sqrt{2}}{2}, -\sqrt{2}, 0 \rangle$$

Section 3.3

#116 $\vec{r}(t) = \langle 2e^t, e^t\cos(t), e^t\sin(t)\rangle$

Soln: $\vec{r}\,'(t) = \langle 2e^t, e^t(\cos(t)-\sin(t)), e^t(\sin(t)+\cos(t))\rangle$

$$= e^t\langle 2, \cos(t)-\sin(t), \sin(t)+\cos(t)\rangle$$

Thus,

$$\|\vec{r}\,'(t)\| = e^t\sqrt{4+(\cos(t)-\sin(t))^2+(\sin(t)+\cos(t))^2}$$

$$= e^t\sqrt{4+\cos^2(t)\cancel{-2\cos(t)\sin(t)}+\sin^2(t)+\sin^2(t)\cancel{+2\sin(t)\cos(t)}+\cos^2(t)}$$

$$= e^t\sqrt{4+1+1} = \sqrt{6}\,e^t$$

Therefore

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\langle 2, \cos(t)-\sin(t), \sin(t)+\cos(t)\rangle}{\sqrt{6}}$$

and compute

$$\vec{T}(0) = \frac{1}{\sqrt{6}}\langle 2, 1, 1\rangle$$

#118 Using answer from #116, compute

$$\vec{T}\,'(t) = \frac{1}{\sqrt{6}}\langle 0, -\sin(t)\ \cos(t), \cos(t)-\sin(t)\rangle$$

$$\|\vec{T}\,'(t)\| = \frac{1}{\sqrt{6}}\sqrt{(-\cos(t)-\sin(t))^2+(\cos(t)-\sin(t))^2}$$

$$= \frac{1}{\sqrt{6}}\sqrt{2[\cos^2(t) \qquad +\sin^2(t)]}$$

$$= \frac{\sqrt{2}}{\sqrt{6}}$$

Thus, $\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{\|\vec{T}\,'(t)\|} = \dfrac{\left(\frac{1}{\sqrt{6}}\right)}{\left(\sqrt{2}/\sqrt{6}\right)}\langle 0, \cos(t)-\sin(t), \cos(t)-\sin(t)\rangle$

So, $\vec{N}(0) = \dfrac{1}{\sqrt{2}}\langle 0, 1-0, 1-0\rangle = \dfrac{1}{\sqrt{2}}\langle 0, 1, 1\rangle$

#132 $\vec{r}(t) = \langle 5\cos(t), 5\sin(t), 0 \rangle$

$\vec{r}'(t) = \langle -5\sin(t), 5\cos(t), 0 \rangle$

$\vec{r}''(t) = \langle -5\cos(t), -5\sin(t), 0 \rangle$

$$\vec{r}'(t) \times \vec{r}''(t) = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5s & 5c & 0 \\ -5c & -5s & 0 \end{bmatrix} = \langle 0, 0, 25\sin^2(t) + 25\cos^2(t) \rangle = \langle 0, 0, 25 \rangle$$

Soln: $\kappa(t) = \dfrac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}$

$$= \frac{\|\langle 0,0,25 \rangle\|}{\|\langle -5\sin(t), 5\cos(t), 0 \rangle\|^3}$$

$$= \frac{25}{\left(\sqrt{25\sin^2(t) + 25\cos^2(t)}\right)^3} = \frac{25}{5^3} = \frac{1}{5}$$

#134 $y = \frac{1}{3}x^3$

Soln: Parametrize the curve: $\vec{r}(t) = \langle t, \frac{1}{3}t^3, 0 \rangle$

$\vec{r}'(t) = \langle 1, t^2, 0 \rangle$

$\vec{r}''(t) = \langle 0, 2t, 0 \rangle$

$$\vec{r}'(t) \times \vec{r}''(t) = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & t^2 & 0 \\ 0 & 2t & 0 \end{bmatrix} = \langle 0, 0, 2t \rangle$$

$$\kappa(t) = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3} = \frac{2t}{\left(\sqrt{1+t^4}\right)^3}$$

Therefore, curvature at $x = t = 1$ is given by

$$\kappa(1) = \frac{2}{\left(\sqrt{1+1^4}\right)^3} = \frac{2}{(\sqrt{2})^3} = \frac{1}{\sqrt{2}}$$

Section 4.1:

#7 $f(x,y) = 4\ln(y^2 - x)$

Soln: Domain is set of points $(x,y) \in \mathbb{R}^2$ such that

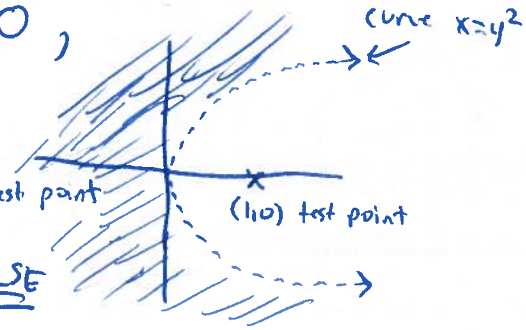
$$y^2 - x > 0,$$

i.e.

$$x < y^2$$

$\downarrow$ plug in test point

$1 < 0^2 \to$ FALSE



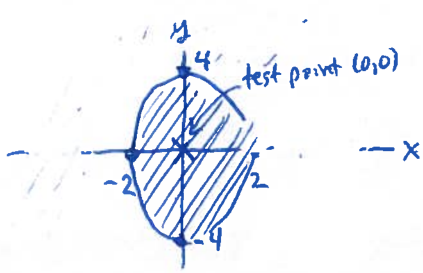
#8 $g(x,y) = \sqrt{16 - 4x^2 - y^2}$

Soln: Domain is set of points $(x,y) \in \mathbb{R}^2$ such that

$$16 - 4x^2 - y^2 \geq 0 \Rightarrow 4x^2 + y^2 \leq 16$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{16} \leq 1$$

ellipse



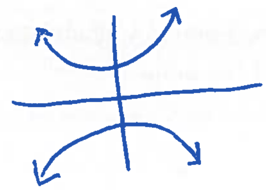
$$\frac{0^2}{4} + \frac{0^2}{16} \leq 1$$

$0 \leq 1$ TRUE

#14) $z(x,y) = y^2 - x^2$, $c = 1$

Soln: Set $z = 1$ to get

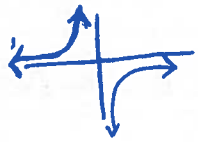
$1 = y^2 - x^2$



#18) $f(x,y) = xy$; $c = 1$, $c = -1$

$c = -1$

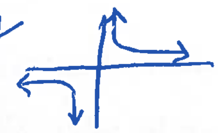
$-1 = xy \to y = -\frac{1}{x}$



$c = +1$

$1 = xy$

$y = \frac{1}{x}$

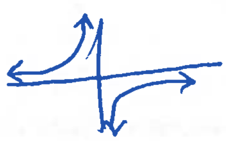


#23) $g(x,y) = e^{xy}$; $c = \frac{1}{2}$, $c = 3$

$c = \frac{1}{2}$

$\frac{1}{2} = e^{xy} \to \ln(\frac{1}{2}) = xy$

$y = \frac{-\ln(2)}{x}$

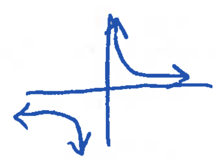


$c = 3$

$3 = e^{xy}$

$\ln(3) = xy$

$y = \frac{\ln(3)}{x}$

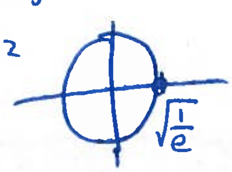


#26) $h(x,y) = \ln(x^2+y^2)$; $c = -1$, $c = 0$, $c = 1$

$c = -1$

$-1 = \ln(x^2+y^2)$

$\frac{1}{e} = x^2+y^2$



$c = 0$

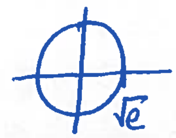
$0 = \ln(x^2+y^2)$

$1 = x^2+y^2$

$c = 1$

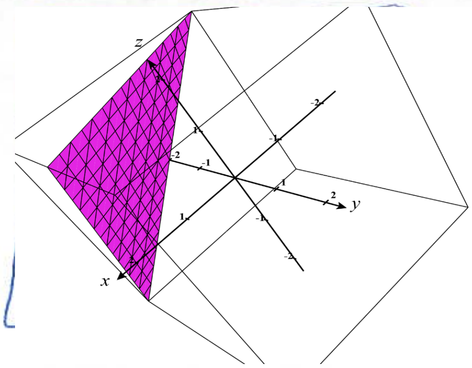
$1 = \ln(x^2+y^2)$

$e = x^2+y^2$



#48 $w(x,y,z) = x - 2y + z \;;\; c = 4$

$4 = x - 2y + z$ ← a plane

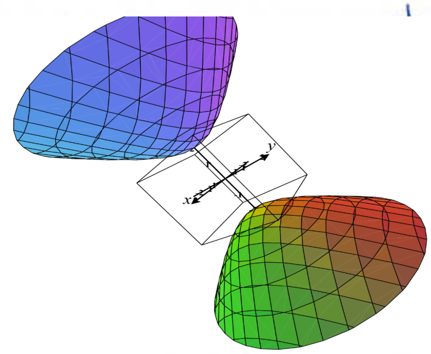


#50 $w(x,y,z) = x^2 + y^2 - z^2 \;;\; c = -4$

$-4 = x^2 + y^2 - z^2$

$\Rightarrow 1 = \frac{x^2}{-4} + \frac{y^2}{-4} + \frac{z^2}{4} \Rightarrow 1 = \frac{z^2}{2^2} - \frac{x^2}{2^2} - \frac{y^2}{2^2}$

hyperboloid in 2 sheets



#56 $E(x,y,z) = \frac{k}{\sqrt{x^2+y^2}} \overset{\text{let } k=1}{=} \frac{1}{\sqrt{x^2+y^2}}$

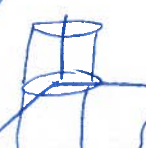
E = 10

$10 = \frac{1}{\sqrt{x^2+y^2}} \Rightarrow 10\sqrt{x^2+y^2} = 1$

$\Rightarrow x^2+y^2 = \frac{1}{100}$ (z free)

$\Downarrow$

($x^2+y^2=r^2$ is circle) → circular cylinder in z-dir



E = 100

$100 = \frac{1}{\sqrt{x^2+y^2}}$

$\Rightarrow$ $x^2+y^2 = \frac{1}{10000}$ (z free)

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Problem A

$$\vec{r}(t) = \langle \cos(t), \sin(t), 1 \rangle$$

$$\vec{r}\,'(t) = \langle -\sin t, \cos t, 0 \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{\sin^2(t) + \cos^2(t)} = \sqrt{1} = 1$$

Thus, $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \langle -\sin(t), \cos(t), 0 \rangle$

$$\vec{T}\,'(t) = \langle -\cos(t), -\sin(t), 0 \rangle$$

$$\|\vec{T}\,'(t)\| = \sqrt{\cos^2(t) + \sin^2(t)} = \sqrt{1} = 1$$

Thus, $\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{\|\vec{T}\,'(t)\|} = \langle -\cos(t), -\sin(t), 0 \rangle$

Therefore, $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t) = \langle -\sin(t), \cos(t), 0 \rangle \times \langle -\cos(t), -\sin(t), 0 \rangle$

$$= \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -s(t) & c(t) & 0 \\ -c(t) & -s(t) & 0 \end{bmatrix}$$

$$= \langle 0, 0, \sin^2(t) + \cos^2(t) \rangle$$

$$= \langle 0, 0, 1 \rangle$$