HW5 MATH 3503 Fall 2018

Section 3.7 **#364** Convert $(r,\theta,z)=(3,\frac{\pi}{3},5)$ into rectangular coords.

Soln: Using Theorem 2.15, rectangular coords are given by

$$\begin{cases} x = 3\cos(\frac{\pi}{3}) = \frac{3}{2} \\ y = 3\sin(\frac{\pi}{3}) = 3(\frac{\sqrt{3}}{2}) \\ z = 5 \end{cases}$$

So, $(x,y,z) = (\frac{3}{2}, \frac{3\sqrt{3}}{2}, 5)$.

#368 Convert $(x,y,z)=(1,1,5)$ to cylindrical coords.

Soln: Using Theorem 2.15,

$$\begin{cases} r = \sqrt{1^2+1^2} = \sqrt{2} \\ \tan(\theta) = \frac{1}{1} \Rightarrow \theta = \arctan(1) = \frac{\pi}{4} \\ z = 5 \end{cases}$$

So, $(r,\theta,z) = (\sqrt{2}, \frac{\pi}{4}, 5)$.

#372 Convert $z = r^2\cos^2(\theta)$ into rectangular.

Soln: Since $x = r\cos(\theta)$, we see $x^2 = r^2\cos^2(\theta)$, so we get

$z = x^2$.

#382 Convert to cylindrical: $y = 2x^2$

Soln: Since $\left.\begin{cases} x = r\cos(\theta) \rightarrow x^2 = r^2\cos^2(\theta) \\ y = r\sin(\theta) \end{cases}\right\}$, so we get

$$r\sin(\theta) = 2r^2\cos^2(\theta) \rightarrow \frac{1}{2}\frac{\sin(\theta)}{\cos^2(\theta)} = r$$

$$\rightarrow r = \frac{1}{2}\tan(\theta)\sec(\theta)$$

#386 Convert $(\rho, \theta, \phi) = (1, \frac{\pi}{6}, \frac{\pi}{6})$ to rectangular coords.

Soln: Using Theorem 2.16,

$$\begin{cases} x = 1\sin(\frac{\pi}{6})\cos(\frac{\pi}{6}) = 1(\frac{1}{2})(\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{4} \\ y = 1\sin(\frac{\pi}{6})\sin(\frac{\pi}{6}) = 1(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4} \\ z = 1\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} \end{cases},$$

So $(x, y, z) = (\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2})$.

#392 Convert $(x, y, z) = (-2, 2\sqrt{3}, 4)$ to spherical.

Soln: Using Theorem 2.16,

$$\rho = \sqrt{(-2)^2 + (2\sqrt{3})^2 + 4^2} = \sqrt{4 + 12 + 16} = \sqrt{32} = 4\sqrt{2}$$

$$\tan(\theta) = \frac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \theta = \arctan(-\sqrt{3}) + \pi$$
$$= -\frac{\pi}{3} + \pi$$
$$= \frac{2\pi}{3}$$

$(x, y) = (-2, 2\sqrt{3})$ is in QII $\Downarrow$ need "$+\pi$" when taking arctan

$$\phi = \arccos\left(\frac{4}{4\sqrt{2}}\right) = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4},$$

So $(\rho, \theta, \phi) = (4\sqrt{2}, \frac{2\pi}{3}, \frac{\pi}{4})$.

Section 3.1 #8 $\lim_{t\to 4} \left\langle \sqrt{t-3}, \frac{\sqrt{t}-2}{t-4}, \tan\left(\frac{\pi}{t}\right) \right\rangle$

$$= \left\langle \lim_{t\to 4} \sqrt{t-3}, \lim_{t\to 4} \frac{\sqrt{t}-2}{t-4}, \lim_{t\to 4} \tan\left(\frac{\pi}{t}\right) \right\rangle$$

$\downarrow$ 1 $\downarrow$ $\frac{0}{0} \Rightarrow$ more work to do $\downarrow$ 1

Note that

calc 1 technique

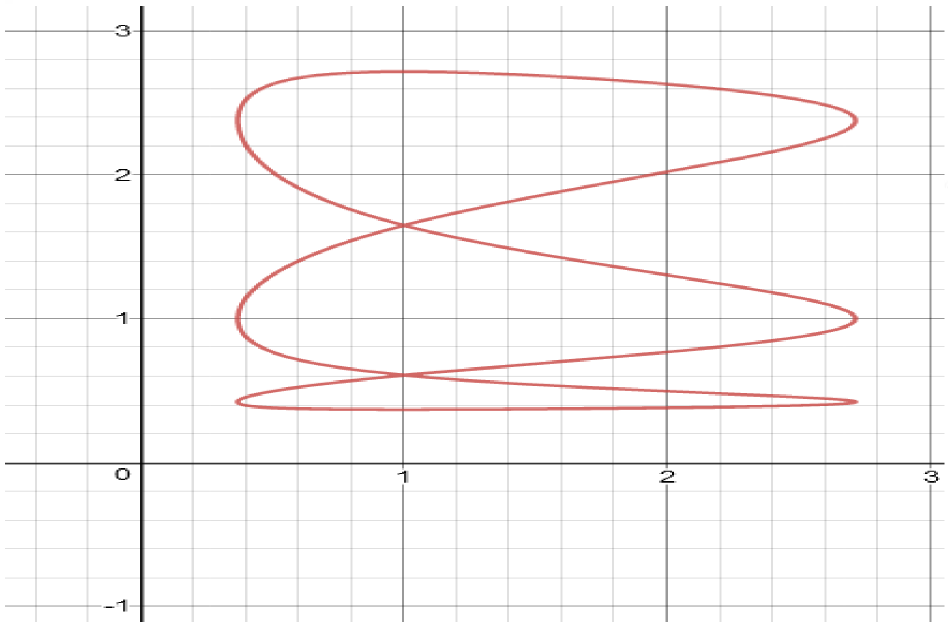
$$\frac{\sqrt{t}-2}{t-4} = \frac{\sqrt{t}-2}{t-4}\left(\frac{\sqrt{t}+2}{\sqrt{t}+2}\right) = \frac{\cancel{(t-4)}}{\cancel{(t-4)}(\sqrt{t}+2)} = \frac{1}{\sqrt{t}+2},$$

thus

$$\lim_{t\to 4} \frac{\sqrt{t}-2}{t-4} = \lim_{t\to 4} \frac{1}{\sqrt{t}+2} = \frac{1}{4}$$

Thus the desired limit resolves to $\langle 1, \frac{1}{4}, 1 \rangle$.

#28



Section 3.2 #42 Compute derivative of

$\vec{r}(t) = \sin(t)\,\vec{i} + \cos(t)\,\vec{j} + e^t\,\vec{k}$

Soln: Compute

$$\frac{d}{dt}\vec{r}(t) = \frac{d}{dt}\langle \sin(t), \cos(t), e^t \rangle$$

$$= \left\langle \frac{d}{dt}\sin(t), \frac{d}{dt}\cos(t), \frac{d}{dt}e^t \right\rangle$$

$$= \langle \cos(t), -\sin(t), e^t \rangle$$

#52 Compute derivative of $\vec{r}(t) = 3t^3\,\vec{i} + 2t^2\,\vec{j} + \frac{1}{t}\,\vec{k}$; $t=1$

Soln: Compute

$$\vec{r}\,'(t) = \left\langle \frac{d}{dt}3t^3, \frac{d}{dt}2t^2, \frac{d}{dt}\frac{1}{t} \right\rangle$$

$$= \langle 9t^2, 4t, -\frac{1}{t^2} \rangle$$

Therefore,

$$\vec{r}\,'(1) = \langle 9, 4, -1 \rangle$$

#56 Find unit tangent vector of $\vec{r}(t) = \cos(t)\,\vec{i} + \sin(t)\,\vec{j} + \sin(t)\,\vec{k}$

Soln: $$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\langle -\sin(t), \cos(t), \cos(t) \rangle}{\sqrt{(-\sin(t))^2 + \cos^2(t) + \cos^2(t)}} = \left\langle \frac{-\sin(t)}{\sqrt{1+\cos^2(t)}}, \frac{\cos(t)}{\sqrt{1+\cos^2(t)}}, \frac{\cos(t)}{\sqrt{1+\cos^2(t)}} \right\rangle$$