MATH 3503 Fall 2018 HW2

#2 $P=(-1,3)$, $R=(-3,7)$, $Q=(1,5)$

$$\vec{PR} = \langle -3,7\rangle - \langle -1,3\rangle = \langle -2,4\rangle$$

#8 $\vec{PQ} = \langle 1,5\rangle - \langle -1,3\rangle = \langle 2,2\rangle$

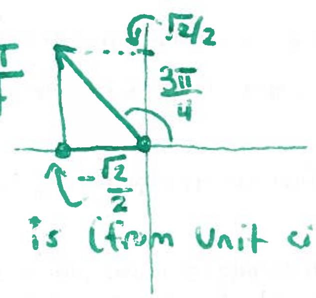
So, $$2\vec{PQ} + \tfrac{1}{2}\vec{PR} = 2\langle 2,2\rangle + \tfrac{1}{2}\langle -2,4\rangle$$
$$= \langle 4,4\rangle + \langle -1,2\rangle$$
$$= \langle 3,6\rangle$$

#10 $\|\vec{PR}\| = \sqrt{(-2)^2+4^2} = \sqrt{20}$

So requested unit vector is

$$\vec{u} = \frac{\vec{PR}}{\|\vec{PR}\|} = \frac{\langle -2,4\rangle}{\sqrt{20}} = \left\langle \frac{-2}{\sqrt{20}}, \frac{4}{\sqrt{20}} \right\rangle$$

#34 Given: $\|\vec{u}\| = 50$, $\theta = \frac{3\pi}{4}$



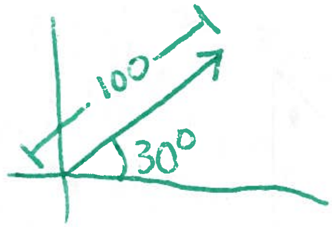
Unit vector in this direction is (from unit circle)

$$\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

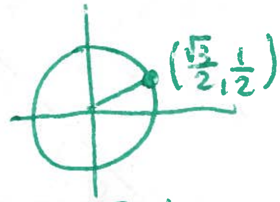
Therefore

$$\vec{u} = 50\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle = \langle -25\sqrt{2}, 25\sqrt{2}\rangle$$

#46) $\theta = 30^\circ$, $\|\vec{v}_0\| = 100$



unit vector in this direction found by unit circle

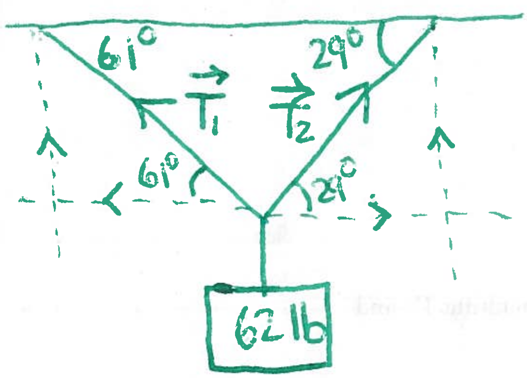


$\Rightarrow$ unit vector is $\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

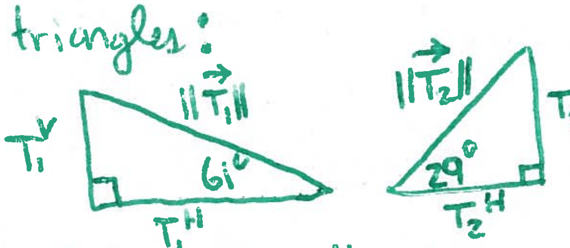
Therefore,

$$\vec{V}_0 = 100 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle = \langle 50\sqrt{3}, 50 \rangle$$

#56)



Two triangles:



"resultant force is zero"

$\Downarrow$

$$(\star)\begin{cases} T_1^H = T_2^H \\ T_1^V + T_2^V = 62 \end{cases}$$

Trigonometry $\Rightarrow$ $T_1^V = \|\vec{T}_1\| \sin(61^\circ)$, $T_1^H = \|\vec{T}_1\| \cos(61^\circ)$

$T_2^V = \|\vec{T}_2\| \sin(29^\circ)$, $T_2^H = \|\vec{T}_2\| \cos(29^\circ)$

Therefore, $(\star)$ becomes

$$\begin{cases} \|T_1\| \cos(61^\circ) = \|T_2\| \cos(29^\circ) & (i) \\ \|T_1\| \sin(61^\circ) + \|T_2\| \sin(29^\circ) = 62 & (ii) \end{cases}$$

From (i) $\rightarrow$

$$(\star\star) \quad \|T_1\| = \|T_2\| \frac{\cos(29^\circ)}{\cos(61^\circ)}$$

Plug that into (ii):

$$\left(\frac{\cos(29^\circ)}{\cos(61^\circ)}\sin(61^\circ) + \sin(29^\circ)\right)\|T_2\| = 62,$$

therefore,

$$\|T_2\| = \frac{62}{\left(\frac{\cos(29^\circ)}{\cos(61^\circ)}\sin(61^\circ) + \sin(29^\circ)\right)}$$

$$\approx 30.06$$

Thus, by (★★),

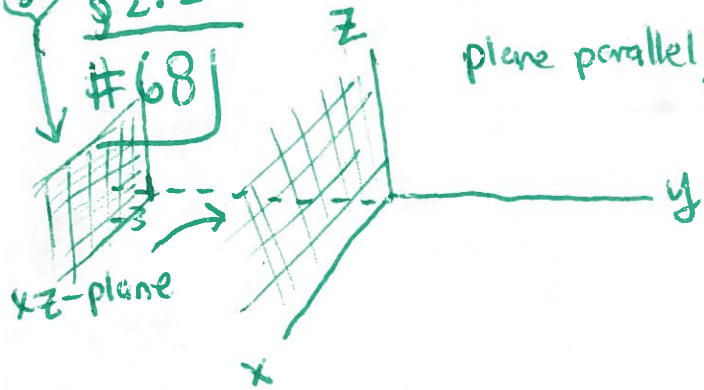
$$\|T_1\| = \|T_2\| \frac{\cos(29^\circ)}{\cos(61^\circ)} \approx (30.06)\left(\frac{\cos(29^\circ)}{\cos(61^\circ)}\right)$$

$$\approx 54.23$$

y=-3

§2.2

#68



plane parallel to xz-plane containing (1,-3,2) is the plane $y=-3$

#84 $\vec{a} = \langle 3,-2,4\rangle$, $\vec{b} = \langle -5,6,-9\rangle$

$$\vec{a}+\vec{b} = \langle -2,4,-5\rangle$$

$$4\vec{a} = \langle 12,-8,16\rangle$$

$$-5\vec{a}+3\vec{b} = -5\langle 3,-2,4\rangle + 3\langle -5,6,-9\rangle$$

$$= \langle -15,10,-20\rangle + \langle -15,18,-27\rangle$$

$$= \langle -30,28,-47\rangle$$

#122 $\vec{r}(t) = \langle t, 2t^2, 4t^2 \rangle$

a) instantaneous velocity: $\vec{v}(t) = \vec{r}\,'(t) = \langle 1, 4t, 8t \rangle$

speed: $\|\vec{v}(t)\| = \sqrt{1+16t^2+64t^2}$

$= \sqrt{1+80t^2}$

acceleration: $\vec{a}(t) = \vec{v}\,'(t) = \langle 0, 4, 8 \rangle$

At time $t=2$:

velocity: $\vec{v}(2) = \langle 1, 8, 16 \rangle$

speed: $\|\vec{v}(2)\| = \sqrt{321} \approx 17.916$

accel: $\vec{a}(2) = \langle 0, 4, 8 \rangle$

§ 2.3

#134 $\vec{u} = 5\vec{i} = \langle 5, 0 \rangle$, $\vec{v} = -6\vec{i} + 6\vec{j} = \langle -6, 6 \rangle$

$72 = 2^3 \cdot 3^2$ ($72 = 2 \cdot 36$, $36 = 2 \cdot 18$)

$\|v\| = \sqrt{36+36} = \sqrt{72}$
$= 6\sqrt{2}$

a) Find angle between:

$$\underbrace{-30 = 5(-6) + 0(6)}_{\text{using definition of dot product}} = \vec{u} \cdot \vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos(\theta) = (5)(\sqrt{72})\cos(\theta)$$

$\Downarrow$

$-30 = 5\sqrt{72}\cos(\theta)$

$\Downarrow$

$$\cos(\theta) = \frac{-30}{5\sqrt{72}} = \frac{-30}{30\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$\Downarrow$

$$\theta = \arccos\left(-\frac{30}{5\sqrt{72}}\right) = \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4} \text{ radians}$$

≈ 2.356 rad

b) θ is <u>not</u> acute since $\frac{3\pi}{4} > \frac{2\pi}{4} = \frac{\pi}{2}$

5

#142) $\vec{a} = \langle x, y \rangle$, $\vec{b} = \langle -y, x \rangle$

$$\vec{a} \cdot \vec{b} = \langle x, y \rangle \cdot \langle -y, x \rangle$$

$$= -xy + xy = 0$$

Therefore, $\vec{a}$ and $\vec{b}$ are orthogonal (See Thm 2.5).

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#170) $\vec{u} = \langle 4, 4, 0 \rangle$, $\vec{v} = \langle 0, 4, 1 \rangle$

a) $$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \vec{u}$$

$\|\vec{u}\| = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$

$\|\vec{u}\|^2 = 32$

$$= \frac{16}{32} \vec{u}$$

$$= \frac{1}{2} \langle 4, 4, 0 \rangle$$

$$= \langle 2, 2, 0 \rangle$$

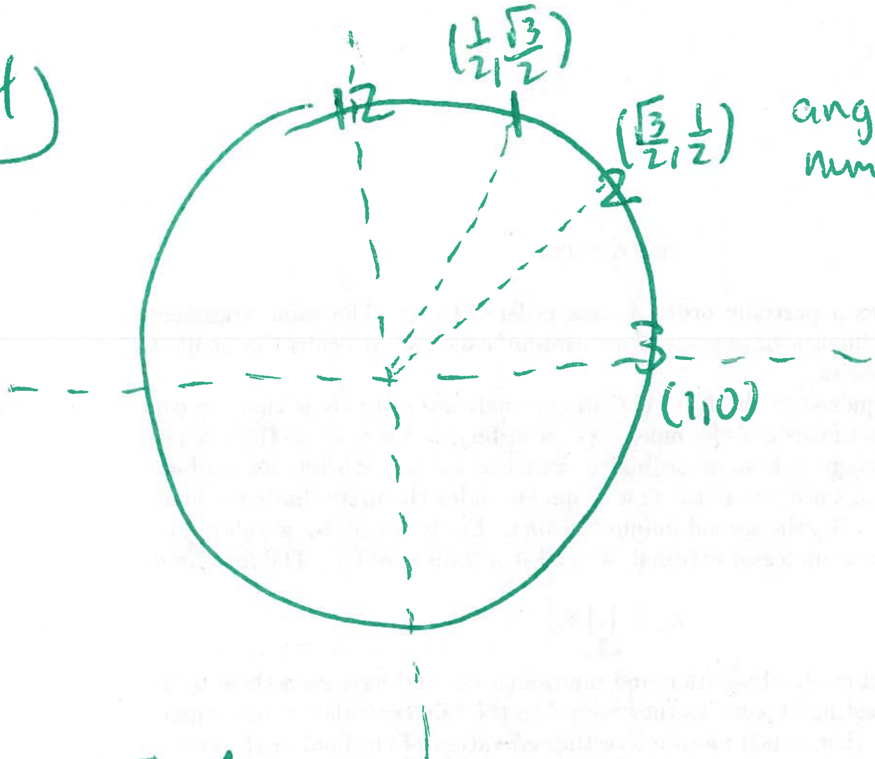
b) $$\text{comp}_{\vec{u}} \vec{v} = \|\text{proj}_{\vec{u}} \vec{v}\| = \| \langle 2, 2, 0 \rangle \|$$

$$= \sqrt{2^2 + 2^2 + 0^2}$$

$$= \sqrt{8}$$

#174)

6



angle between numbers: $\frac{2\pi}{12} = \frac{\pi}{6}$

Therefore

1 is $\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

2 is $\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$, and

3 is $\langle 1, 0 \rangle$