HW14 MATH 3503 Fall 2018

#275 $2x-4y+3z=16$

$\Downarrow$

$z=\frac{16-2x+4y}{3} \Rightarrow$ Parametrization $\vec{r}(u,v)=\langle u,v,\frac{16-2u+4v}{3}\rangle$

#282 already posted to site under "Notes"

#284 $\vec{F}=\langle x,2y,-3z\rangle$

S : part of plane $15x-12y+3z=6 \Rightarrow$ Parametrize $z=\frac{6-15x+12y}{3}$

above $\{0\le x\le 1,\ 0\le y\le 1\}$

$=2-5x+4y$

$\Downarrow$

$\left\{\begin{array}{l}\vec{r}(u,v)=\langle u,v,2-5u+4v\rangle\\ 0\le u\le 1,\ 0\le v\le 1\end{array}\right\}$

$r_u=\langle 1,0,-5\rangle$

$r_v=\langle 0,1,4\rangle$

$\Downarrow$

$r_u\times r_v=\langle 5,-4,1\rangle$

Therefore we compute

$$\iint_S \vec{F}\cdot d\vec{r}=\int_0^1\int_0^1 \vec{F}(\vec{r}(u,v))\cdot(r_u\times r_v)\,du\,dv$$

$$=\int_0^1\int_0^1 \langle \overset{x}{u},\overset{2y}{2v},\overset{-3z}{-3(2-5u+4v)}\rangle\cdot\langle 5,-4,1\rangle\,du\,dv$$

$$=\int_0^1\int_0^1 \underbrace{5u-8v-6+15u-12v}_{=20u-20v-6}\,du\,dv$$

$$=\int_0^1 10u^2-20vu-6u\Big|_{u=0}^{u=1}dv=\int_0^1 10-20v-6\,dv$$

$$=\int_0^1 -20v+4\,dv$$

$$=-10v^2+4v\Big|_0^1=-10+4=-6$$