HW10 MATH 3503 Fall 2018



Section 5.1 #14

$$\int_0^2\int_0^1 x+2e^y-3 \; dx\, dy = \int_0^1\int_0^2 x+2e^y-3\, dy\, dx$$

$$= \int_0^1 \underbrace{xy+2e^y-3y\Big|_{y=0}^{y=2}}_{=(2x+2e^2-6)-(0+2-0)} dx$$

$$= \int_0^1 2x+2e^2-8\,dx$$

$$= x^2+2e^2x-8x\Big|_0^1$$

$$= 1+2e^2-8$$

$$= 2e^2-7$$

#20 $\displaystyle\int_1^9\int_4^2 \frac{\sqrt{x}}{y^2}\, dy\, dx = \int_4^2\int_1^9 x^{1/2}y^2\, dx\, dy$

$$= \int_4^2 \frac{x^{3/2}}{3/2}\, y^2\Big|_{x=1}^{x=9} dy$$

$$= \frac{2}{3}\int_4^2 (9^{3/2}-1^{3/2})y^2\, dy$$

$$= \frac{2}{3}(3^3-1)\int_4^2 y^2\, dy$$

$$= \frac{2}{3}(26)\,\frac{y^3}{3}\Big|_4^2$$

$$= \frac{52}{9}(2^3-4^3)$$

#22

$$\int_{\pi/12}^{\pi/8}\int_{\pi/4}^{\pi/3} \cot(x)+\tan(2y)\,dx\,dy = \int_{\pi/12}^{\pi/8} \ln(\sin(x))+x\tan(2y)\Big|_{x=\pi/4}^{x=\pi/3} dy$$

$$\int \cot(x)\,dx = \int \frac{\cos(x)}{\sin(x)}\,dx \overset{\substack{du=\cos(x)dx\\ u=\sin x}}{=} \int \frac{1}{u}\,du$$

$$= \ln(u)$$

$$= \ln(\sin(x))$$

$$\int \tan(x)\,dx = \int \frac{\sin x}{\cos x}\,dx \overset{\substack{du=-\sin x\,dx\\ u=\cos x}}{=} -\int \frac{1}{u}\,du$$

$$= -\ln(\cos(x))$$

$$= \int_{\pi/12}^{\pi/8}\Big[\ln(\sin(\pi/3))+(\pi/3)\tan(2y)\Big]-\Big[\ln(\sin(\pi/4))+\pi/4\tan(2y)\Big]dy$$

$$= \int_{\pi/12}^{\pi/8} \underbrace{\ln\left(\frac{\sqrt{3}}{2}\right)-\ln\left(\frac{\sqrt{2}}{2}\right)}_{\substack{=\ln\left(\frac{\sqrt{3}/2}{\sqrt{2}/2}\right)\\ =\ln(\sqrt{3/2})}} dy + \underbrace{\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}_{=\frac{\pi}{12}}\int_{\pi/12}^{\pi/8}\tan(2y)\,dy$$

$u=2y$

$\frac{1}{2}du=dy$

$$= \ln\left(\sqrt{\frac{3}{2}}\right)\int_{\pi/12}^{\pi/8} 1\,dy + \frac{\pi}{12}\cdot\frac{1}{2}\int_{\pi/6}^{\pi/4}\tan(u)\,du$$

$$= \ln\left(\sqrt{3/2}\right)\left[\frac{\pi}{8}-\frac{\pi}{12}\right]+\frac{\pi}{24}\left(-\ln(\cos(u))\right)\Big|_{\pi/6}^{\pi/4}$$

$$= \ln\left(\sqrt{3/2}\right)\left[\frac{\pi}{24}\right]-\frac{\pi}{24}\underbrace{\left(\ln(\cos(\pi/4))-\ln(\cos(\pi/6))\right)}$$

$$= \ln(\sqrt{2}/2)-\ln(\sqrt{3}/2)$$

$$= \ln\left(\frac{\sqrt{2}/2}{\sqrt{3}/2}\right) = \ln\left(\sqrt{\frac{2}{3}}\right)$$

$$= \ln\left(\sqrt{\frac{3}{2}}\right)\frac{\pi}{24} - \ln\left(\sqrt{\frac{2}{3}}\right)\frac{\pi}{24}$$

$$= \frac{\pi}{24}\ln\left(\frac{\sqrt{3/2}}{\sqrt{2/3}}\right) = \frac{\pi}{24}\ln\left(\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{3}}{\sqrt{2}}\right) = \frac{\pi}{24}\ln\left(\frac{3}{2}\right)$$

#24

$$\int_1^e\int_1^e \frac{\sin(\ln(x))\cos(\ln(y))}{xy}\,dx\,dy \overset{\substack{u=\ln(x)\\ du=\frac{1}{x}dx}}{=} \int_1^e\int_0^1 \sin(u)\cdot\frac{\cos(\ln(y))}{y}\,du\,dy$$

$$= \int_1^e (-\cos(u))\left(\frac{\cos(\ln(y))}{y}\right)\Bigg|_{u=0}^{u=1} dy$$

$$= \left(-\cos(1)-(-\cos(0))\right)\int_1^e \frac{\cos(\ln(y))}{y}\,dy$$

$v=\ln(y)$
$dv=\frac{1}{y}dy$

$$= (1-\cos(1))\int_0^1 \cos(v)\,dv$$

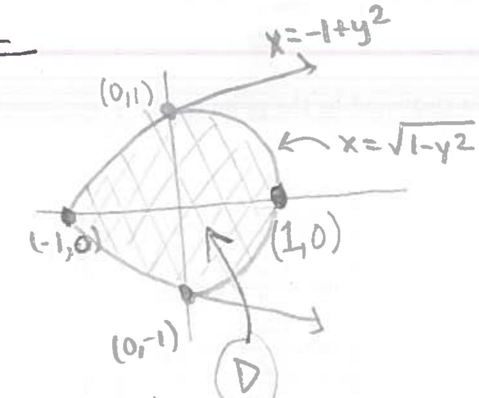
$$= (1-\cos(1))\left[\sin(v)\Big|_0^1\right.$$

$$= (1-\cos(1))(\sin(1)-0)$$

$$= \sin(1)(1-\cos(1))$$

Section 5.2

#67




$$\iint_D f(x,y)\,dA = \int_{-1}^1\int_{-1+y^2}^{\sqrt{1-y^2}} xy+1\,dx\,dy = \int_{-1}^1 \frac{x^2y}{2}+x\Bigg|_{x=-1+y^2}^{x=\sqrt{1-y^2}} dy$$

$$= \int_{-1}^1\left[\left(\frac{(1-y^2)y}{2}+\sqrt{1-y^2}\right)-\left(\frac{\overbrace{(-1+y^2)^2y}^{(y^4-2y^2+1)y}}{2}+(-1+y^2)\right)\right]dy$$

$$= \int_{-1}^1 \frac{-y^3}{2}+\cancel{\frac{y}{2}}+\sqrt{1-y^2}-\frac{y^5}{2}+\frac{2y^3}{2}-\cancel{\frac{y}{2}}+1-y^2\,dy$$

$$= \int_{-1}^1 \frac{y^3}{2}-\frac{y^5}{2}-y^2+1\,dy + \int_{-1}^1\sqrt{1-y^2}\,dy$$

$\theta=\arcsin(y)$

trig sub
$y=\sin(\theta)$
$dy=\cos(\theta)\,d\theta$

$$= \left[\frac{y^4}{8}-\frac{y^6}{12}-\frac{y^3}{3}+y\Bigg|_{-1}^1\right. + \int_{-\pi/2}^{\pi/2}\underbrace{\sqrt{1-\sin^2(\theta)}}_{=\sqrt{\cos^2(\theta)}}\cos(\theta)\,d\theta$$

$$= \left[\left(\cancel{\frac{1}{8}} - \cancel{\frac{1}{12}} - \frac{1}{3} + 1\right) - \left(\cancel{\frac{1}{8}} - \cancel{\frac{1}{12}} + \frac{1}{3} - 1\right)\right] + \int_{-\pi/2}^{\pi/2} \underbrace{\cos^2(\theta)}_{=\frac{1+\cos(2\theta)}{2}} d\theta$$

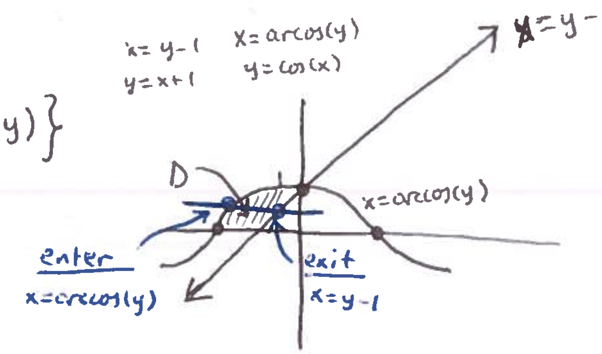
$$= \left(-\frac{2}{3} + 2\right) + \int_{-\pi/2}^{\pi/2} \frac{1}{2}\, d\theta + \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos(2\theta)\, d\theta$$

$$= \frac{4}{3} + \frac{1}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) + \underbrace{\frac{1}{4}\sin(2\theta)\Big|_{-\pi/2}^{\pi/2}}_{=0}$$

$$= \frac{4}{3} + \frac{\pi}{2}$$

**#76** $f(x,y) = 2$

$$D = \{(x,y) : 0 \le y \le 1,\ y-1 \le x \le \arccos(y)\}$$




Looked up

$$\int \arccos(y)\,dy = y\arccos(y) - \sqrt{1-y^2} + C$$

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{y-1}^{\arccos(y)} 2\, dx\, dy$$

$$= 2\int_0^1 \arccos(y) \overbrace{-(y-1)}^{-y+1}\, dy$$

$$= 2\left[y\arccos(y) - \sqrt{1-y^2} - \frac{y^2}{2} + y\right|_0^1$$

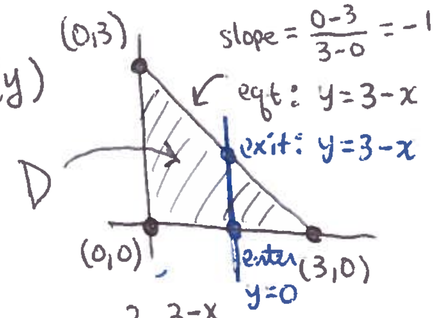
$$= 2\left[\left(\underbrace{1\arccos(1)}_{=0} - \sqrt{1-1^2} - \frac{1^2}{2} + 1\right) - \left(0 - \sqrt{1-0} - 0 + 0\right)\right]$$

$$= 2\left[\left(0 - 0 - \frac{1}{2} + 1\right) - (-1)\right]$$

$$= 2\left(\frac{3}{2}\right)$$

$$= 3$$

**#78** $f(x,y) = \sin(y)$




$$\iint_D \sin(y)\,dA = \int_0^3 \int_0^{3-x} \sin(y)\,dy\,dx$$

$$= \int_0^3 -\cos(y)\Big|_{y=0}^{y=3-x} dx$$

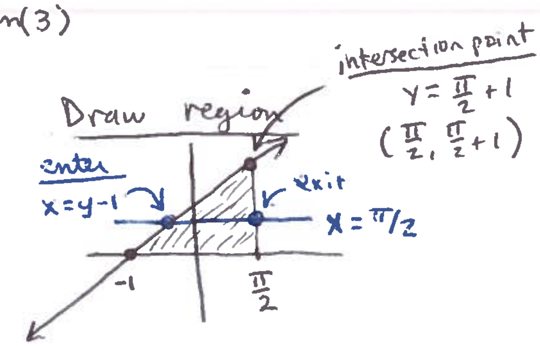
$$= \int_0^3 -\cos(3-x)+1\;dy$$

$$= \sin(3-x)+x\Big|_0^3$$

$$= (\sin(0)+3) - (\sin(3)+0)$$

$$= 3-\sin(3)$$

**#96** $\displaystyle\int_{-1}^{\pi/2}\int_{0}^{x+1} \sin(x)\,dy\,dx$ $\quad x+1 \leftarrow$ exit $y=x+1$ $\quad 0 \leftarrow$ enter $y=0$




Integral in opposite order:

$$\int_{-1}^{\pi/2}\int_{0}^{x+1} \sin(x)\,dy\,dx = \int_0^{\frac{\pi}{2}+1}\int_{y-1}^{\pi/2} \sin(x)\,dx\,dy$$

$$= \int_0^{\pi/2+1} -\cos(x)\Big|_{x=y-1}^{x=\pi/2} dy$$

$$= \int_0^{\pi/2+1} \underbrace{-\cos(\pi/2)}_{=0} - (-\cos(y-1))\,dy$$

$$= \int_0^{\pi/2+1} \cos(y-1)\,dy$$

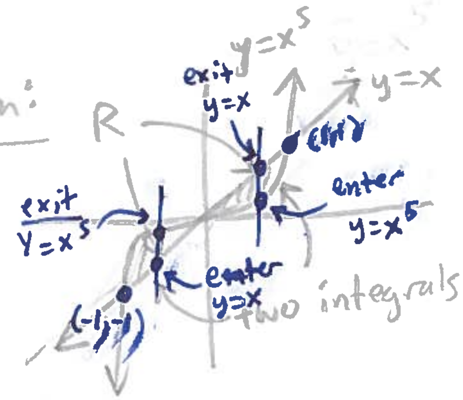
$$= \sin(y-1)\Big|_0^{\pi/2+1}$$

$$= \sin(\tfrac{\pi}{2}+1-1) - \sin(0-1)$$

$$= 1 - \sin(-1)$$

$$= 1+\sin(1)$$

#102 $f(x,y)=2x+y^2$ Region:




6

$$\iint_R f(x,y)\,dA$$

$$= \int_{-1}^{0}\int_{x}^{x^5} 2x+y^2\,dy\,dx + \int_{0}^{1}\int_{x^5}^{x} 2x+y^2\,dy\,dx$$

$$= \int_{-1}^{0}\left[2xy+\frac{y^3}{3}\right|_{y=x}^{y=x^5} dx + \int_{0}^{1}\left[2xy+\frac{y^3}{3}\right|_{y=x}^{y=x^5} dx$$

$$= \int_{-1}^{0}\left(2x^6+\frac{x^{15}}{3}\right)-\left(2x^2+\frac{x^3}{3}\right)dx + \int_{0}^{1}\left(2x^6+\frac{x^{15}}{3}\right)-\left(2x^2+\frac{x^3}{3}\right)dx$$

$$= \left[\frac{2x^7}{7}+\frac{x^{16}}{48}-\frac{2x^3}{3}-\frac{x^4}{12}\right|_{-1}^{0} + \left[\frac{2x^7}{7}+\frac{x^{16}}{48}-\frac{2x^3}{3}-\frac{x^4}{12}\right|_{0}^{1}$$

$$= \left[(0)-\left(-\frac{2}{7}+\cancel{\frac{1}{48}}+\frac{2}{3}+\cancel{\frac{1}{12}}\right)\right] + \left[\left(\frac{2}{7}+\cancel{\frac{1}{48}}-\frac{2}{3}-\cancel{\frac{1}{12}}\right)-(0)\right]$$

$$= \frac{4}{7}-\frac{4}{3} = \frac{12}{21}-\frac{28}{21} = -\frac{16}{24} = -\frac{8}{12} = -\frac{4}{6} = -\frac{2}{3}$$