Homework

1. Recall the definition of the Ack function Compute Ack$(1, 2)$.
2. Consider a function defined recursively by

$$x(n+1) = (n+1) * x(n), \quad x(0) = 3.$$

What is $x(4)$? What is $x(5)$? In general, what is the (non-recursive) formula for $x(n)$ (for $n = 0, 1, 2, 3, \ldots$)?

Consider the theory of four-point geometry, defined by the following axioms:

Axiom 1	There exist exactly four points.
Axiom 2	Each two distinct points have exactly one line that contains both of them.
Axiom 3	Each line is on exactly two points.

3. Draw a picture (i.e. "model") of four-point geometry.
4. Show that Axiom 1 of four-point geometry is independent of axioms 2 and 3.
5. Show that Axiom 2 of four-point geometry is independent of axioms 1 and 3.
6. Show that Axiom 3 of four-point geometry is independent of axioms 1 and 2.

the following sequence of formulas is provided to prove that the formula $S0 \cdot SS0 = S0 + S0$ is a theorem of first order arithmetic

Formulas used in proof of HW10 problem 2b
$S0 \cdot S0 = S0$
$S0 \cdot SS0 = (S0 \cdot S0) + S0$
$S0 \cdot SS0 = S0 + S0$

Consider the following assignment of values for symbols:

Symbol in the theory	Assigned numerical value
S	2
0	3
$\cdot$	4
$=$	5
(	6
)	7
$+$	8

8. Find the Gödel number of the formula in each line.
9. Find the super Gödel number of the proof