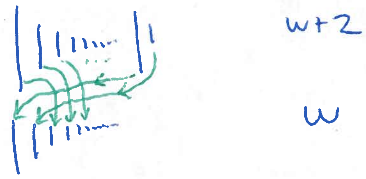
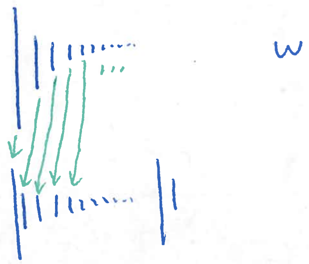
HW 10 MATH 2510 Spring 2018

1) Define $f: \omega+2 \to \omega$ by



This map is one-to-one, showing $|\omega+2| \leq |\omega|$.

Define $g: \omega \to \omega+2$ by



This map is one-to-one, showing $|\omega| \leq |\omega+2|$.

Since $|\omega| \leq |\omega+2|$ and $|\omega+2| \leq |\omega|$, we conclude that $|\omega| = |\omega+2|$.

2) a) subsets of $\{0,1,2,3\}$:

b)

c)

$\cancel{\emptyset}$ $\cancel{\{0\}}$ $\cancel{\{1\}}$ $\cancel{\{2\}}$ $\{3\} \nearrow 1$

$\cancel{\{0,1\}}$ $\cancel{\{0,2\}}$ $\{0,3\} \nearrow 2$ $\cancel{\{1,2\}}$ $\{1,3\} \nearrow 2$

$2 \swarrow \{2,3\}$ $\cancel{\{0,1,2\}}$ $\{0,1,3\} \searrow 3$ $\{0,2,3\} \searrow 3$ $\{1,2,3\} \nearrow 3$

$\{0,1,2,3\} \searrow 4$

d) $cf(\{0,1,2,3\}) = 1$

e) Since $\{\omega\}$ is a cofinal subset of $\omega+1$, $cf(\omega+1) = 1$.

3) a) $2 \oplus 2 = \left|(\{0,1\}\times\{0\}) \cup (\{0,1\}\times\{1\})\right|$

$= \left|\{(0,0),(1,0)\} \cup \{(0,1),(1,1)\}\right|$

$= \left|\{(0,0),(1,0),(0,1),(1,1)\}\right|$

$= 4$

b) $2 \otimes 3 = |2 \times 3|$

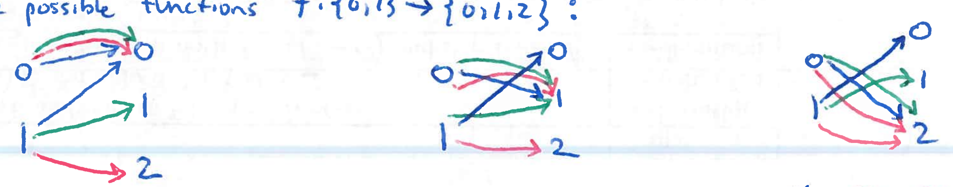
$= \left|\{0,1\} \times \{0,1,2\}\right|$

$= \left|\{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2)\}\right|$

$= 6$

c) $3^2 = \left|{}^2 3\right| = \left|\{f : 2 \to 3\}\right| = \left|\{f : \{0,1\} \to \{0,1,2\}\}\right|$

nine possible functions $f : \{0,1\} \to \{0,1,2\}$:



1 2 3 4 5 6 7 8 9

Thus

$$3^2 = \left|{}^2 3\right| = 9.$$