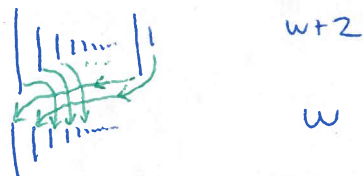
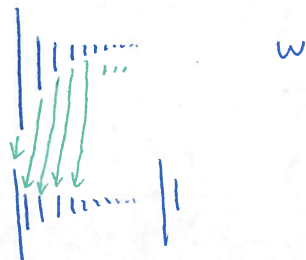


① Define $f: w+2 \rightarrow w$ by



This map is one-to-one, showing $|w+2| \leq |w|$.

Define $g: w \rightarrow w+2$ by



This map is one-to-one, showing $|w| \leq |w+2|$.

Since $|w| \leq |w+2|$ and $|w+2| \leq |w|$, we conclude that $|w| = |w+2|$.

②

- a) subsets of $\{0,1,2,3\}$:
- | | | | | |
|---------------------------------|-----------------------------------|-------------------------------|---------------------------------|-------------|
| $\{0\}$ | $\{1\}$ | $\{2\}$ | $\{3\}$ | $\{3\}$ |
| $\{0,1\}$ | $\{0,2\}$ | $\{0,3\}$ | $\{1,2\}$ | $\{1,3\}$ |
| $\{2,3\}$ | $\{0,1,2\}$ | $\{0,1,3\}$ | $\{0,2,3\}$ | $\{1,2,3\}$ |
| | $\{0,1,2,3\}$ | | | |
- Red arrows indicate cardinalities: $\{3\}$ has 1 element, $\{1,3\}$ has 2, $\{2,3\}$ has 2, $\{0,3\}$ has 2, $\{0,1,3\}$ has 3, $\{0,2,3\}$ has 3, $\{1,2,3\}$ has 3, and $\{0,1,2,3\}$ has 4.

b) $cf(\{0,1,2,3\}) = 1$

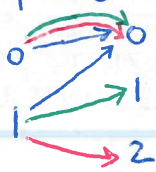
c) since $\{w\}$ is a cofinal subset of $w+1$, $cf(w+1) = 1$.

$$\begin{aligned}
 3) \quad a) \quad 2 \otimes 2 &= |(\{0,1\} \times \{0\}) \cup (\{0,1\} \times \{1\})| \\
 &= |\{(0,0), (1,0)\} \cup \{(0,1), (1,1)\}| \\
 &= |\{(0,0), (1,0), (0,1), (1,1)\}| \\
 &= 4
 \end{aligned}$$

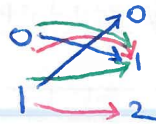
$$\begin{aligned}
 b) \quad 2 \otimes 3 &= |2 \times 3| \\
 &= |\{0,1\} \times \{0,1,2\}| \\
 &= |\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2)\}| \\
 &= 6
 \end{aligned}$$

$$c) \quad 3^2 = |{}^2_3| = |\{f: 2 \rightarrow 3\}| = |\{f: \{0,1\} \rightarrow \{0,1,2\}\}|$$

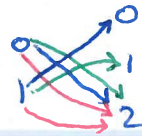
nine possible functions $f: \{0,1\} \rightarrow \{0,1,2\}$:



① ② ③



④ ⑤ ⑥



⑦ ⑧ ⑨

Thus

$$3^2 = |{}^2_3| = 9.$$