

Quiz 9 MATH 2501 FALL 2018

Compute $\int_0^2 x^2 dx$ from the limit definition.

Soln: Here $\Delta x = \frac{2-0}{n} = \frac{2}{n}$

So,

$$\int_0^2 x^2 dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n (0+k\Delta x)^2 \Delta x$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n k^2 \left(\frac{2}{n}\right)^2 \cdot \frac{2}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{2^3}{n^3} \sum_{k=1}^n k^2$$

$$(2n+1)(n+1) = 2n^2 + 2n + n + 1$$

$$= 2n^2 + 3n + 1$$

$$= \lim_{n \rightarrow \infty} \frac{2^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{8}{6} \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{n^3}$$

$$= \frac{8}{6} \lim_{n \rightarrow \infty} \left(2 + \frac{3}{n} + \frac{1}{n^2} \right)$$

$$= \frac{8}{3}$$