like
#237 Section 4.5 (modified)

$f(x) = \cos(x)e^x$, over $[-\pi, \pi]$

$f'(x) = -\sin(x)e^x + \cos(x)e^x = e^x(\cos(x) - \sin(x))$

$f''(x) = e^x(\cos(x) - \sin(x)) + e^x(-\sin(x) - \cos(x)) = -e^x(\sin(x) + \cos(x))$

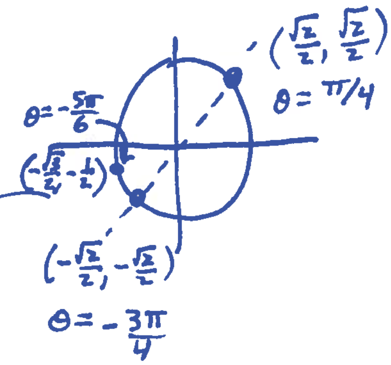
Critical points of f:

$f'(x) = \boxed{e^x(\cos(x) - \sin(x)) \overset{\text{set}}{=} 0}$

↓ divide by e^x

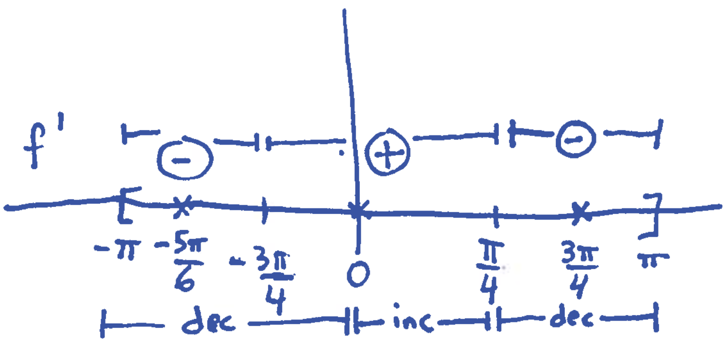
$\cos(x) - \sin(x) = 0$

$\cos(x) = \sin(x)$



$\Downarrow$

$$\boxed{x = \frac{\pi}{4}, -\frac{3\pi}{4}}$$



$$\boxed{\begin{array}{l} \text{local min at } x = -\frac{3\pi}{4} \\ \text{local max at } x = \pi/4 \end{array}}$$

Calculate f' at test pts:

$f'\left(-\frac{5\pi}{6}\right) = \underbrace{e^{-\frac{5\pi}{6}}}_{>0} \underbrace{\left(\cos\left(-\frac{5\pi}{6}\right) - \sin\left(-\frac{5\pi}{6}\right)\right)}$

$= -\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right)$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}$ ← negative

$\approx -0.366 < 0$

neg

$f'(0) = e^0(\cos(0) - \sin(0))$

$= 1(1 - 0) = 1 > 0$

$f'\left(\frac{3\pi}{4}\right) = \underbrace{e^{3\pi/4}}_{>0} \underbrace{\left(\cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right)\right)}$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$

$= -\sqrt{2}$

< 0

Critical pts of f'

$$f''(x) = \boxed{-e^{x}\left(\sin(x)+\cos(x)\right) \overset{\text{set}}{=} 0}$$

$\downarrow$ div by $-e^{x}$

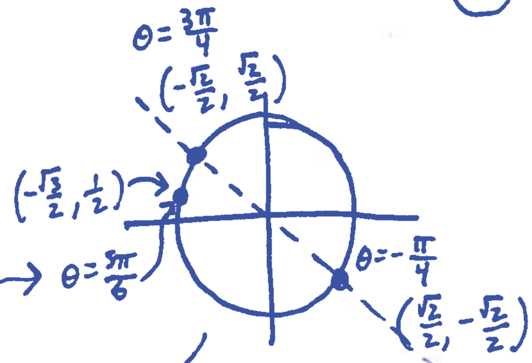
$$\sin(x)+\cos(x)=0$$

$$\downarrow$$

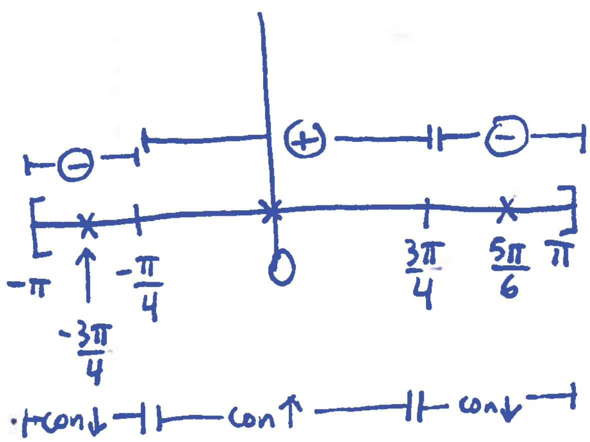
$$\sin(x) = -\cos(x)$$

$$\downarrow$$

$$x = \frac{3\pi}{4},\ -\frac{\pi}{4}$$



inflection points of f



inflection points at $x = -\frac{\pi}{4}$

and at $x = \frac{3\pi}{4}$

Calculate f'' at test pts

$$f''\left(-\frac{3\pi}{4}\right) = \underbrace{e^{-\frac{3\pi}{4}}}_{>0}\underbrace{\left(\sin\left(-\frac{3\pi}{4}\right)+\cos\left(-\frac{3\pi}{4}\right)\right)}_{}$$

$$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$$

$$= -\sqrt{2}$$

negative

$$f''(0) = e^{0}\left(\sin(0)+\cos(0)\right)$$

$$= 1(0+1) > 0$$

$$f''\left(\frac{5\pi}{6}\right) = \underbrace{e^{5\pi/6}}_{>0}\underbrace{\left(\sin\left(\frac{5\pi}{6}\right)+\cos\left(\frac{5\pi}{6}\right)\right)}_{}$$

$$= \underbrace{\frac{1}{2} - \frac{\sqrt{3}}{2}}_{\text{negative}}$$