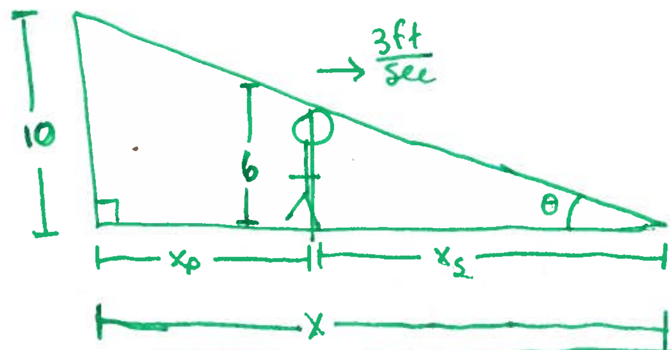
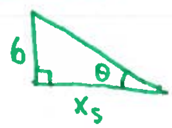
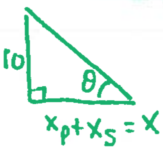
#10



Given: $\frac{dx_p}{dt} = 3 \frac{ft}{sec}$



and



$x_p = 10$

$x = x_p + x_s$

$$\underbrace{\frac{6}{x_s}}_{\text{shorter } \Delta} = \tan(\theta) = \underbrace{\frac{10}{x_p + x_s}}_{\text{taller } \Delta}$$

Seek: $\frac{dx}{dt}$

Solution 1: From $\frac{6}{x_s} = \frac{10}{x_p + x_s}$, we may use algebra to reach

$$6(x_p + x_s) = 10x_s$$

Since we know x_p and are seeking $\frac{dx}{dt}$, we want to eliminate x_s.
Since $x_p + x_s = x$ and $x_s = x - x_p$, we get

$$6x = 10(x - x_p)$$

$$6x = 10x - 10x_p$$

$$10x_p = 4x$$

$\Downarrow$ take $\frac{d}{dt}$

$$10 \frac{dx_p}{dt} = 4 \frac{dx}{dt},$$

thus divide by 4 to get $\frac{10}{4} \frac{dx_p}{dt} = \frac{dx}{dt}$,

or $\boxed{\frac{5}{2} \frac{dx_p}{dt} = \frac{dx}{dt}}$

Now we may calculate

$$\left.\frac{dx}{dt}\right|_{\frac{dx_p}{dt} = 3} = \frac{5}{2}(3) = \frac{15}{2} \frac{ft}{sec}$$

Solution 2: Take

$$\frac{6}{x_s} = \frac{10}{x_p + x_s}$$

and use $x_p + x_s = x$ and $x_s = x - x_p$ to get

$$(\star) \qquad \frac{6}{x - x_p} = \frac{10}{x},$$

and rewrite using negative exponents as

$$6(x - x_p)^{-1} = 10x^{-1}$$

Now differentiate with respect to t and get

$$6\frac{d}{dt}(x - x_p)^{-1} = 10\frac{d}{dt}x^{-1}$$

$$\Big\downarrow \text{ power rule + chain rule}$$

$$(\star\star) \qquad -6\underbrace{(x - x_p)^{-2}}_{=\frac{1}{(x-x_p)^2}}\left[\frac{dx}{dt} - \frac{dx_p}{dt}\right] = -10\underbrace{x^{-2}}_{=\frac{1}{x^2}}\frac{dx}{dt}$$

If we take $x_p = 10$ (from given), then from $(\star)$ we get

$$\frac{6}{x - 10} = \frac{10}{x}$$

$$\downarrow$$

$$6x = 10(x - 10) \rightarrow 100 = 4x$$

$$\rightarrow x = \frac{100}{4} = 25$$

Therefore, plugging $\frac{dx_p}{dt} = 3$, $x_p = 10$, and $x = 25$ into $(\star\star)$ yields

$$\frac{-6}{(25-10)^2}\left[\frac{dx}{dt} - 3\right] = -10 \cdot \left(\frac{1}{25^2}\right)\frac{dx}{dt}$$

$$\frac{-6}{15^2}\left[\frac{dx}{dt}\right] + \frac{18}{15^2} = -\frac{10}{25^2}\frac{dx}{dt}$$

$$\Big\downarrow \text{ multiply by 25} \longrightarrow \frac{25}{15^2} = \frac{\cancel{5^2}}{3^2\cancel{5^2}} = \frac{1}{9}$$

$$-\frac{6}{9}\frac{dx}{dt} + \frac{18}{9} = -\frac{10}{25}\frac{dx}{dt}$$

$$\left(-\frac{6}{9} + \frac{10}{25}\right)\frac{dx}{dt} = -\frac{18}{9}$$

$$-\frac{60}{225}\frac{dx}{dt} = -\frac{18}{9} \rightarrow \frac{dx}{dt} = \frac{(18)(225)}{9(60)} = \frac{(\cancel{2}\cdot\cancel{3^2})(5^2\cdot 3^2)}{\cancel{3^2}(2^2\cdot\cancel{3}\cdot\cancel{5})} = \frac{15}{2}\,\frac{\text{ft}}{\text{sec}}$$

$$\begin{array}{r} 25 \\ -10 \\ \hline 15 \end{array}$$

$$15^2 = 3^2 \cdot 5^2 \qquad \begin{array}{r} 25 \\ \times 6 \\ \hline 150 \end{array} \qquad \begin{array}{r} 25 \\ \times 9 \\ \hline 225 \end{array}$$

$$-\frac{6}{9} + \frac{10}{25} = \frac{-6\cdot 5^2 + 10\cdot 3^2}{3^2\cdot 5^2} = \frac{-150 + 90}{3^2\cdot 5^2} = -\frac{60}{225}$$

the same answer!!!