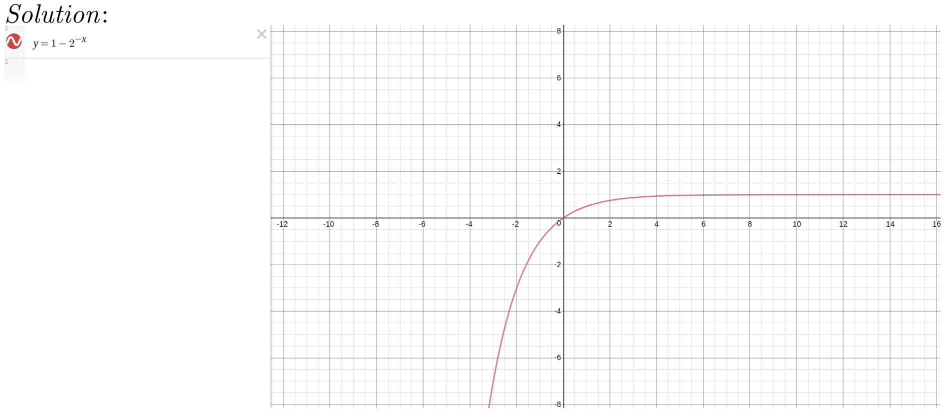
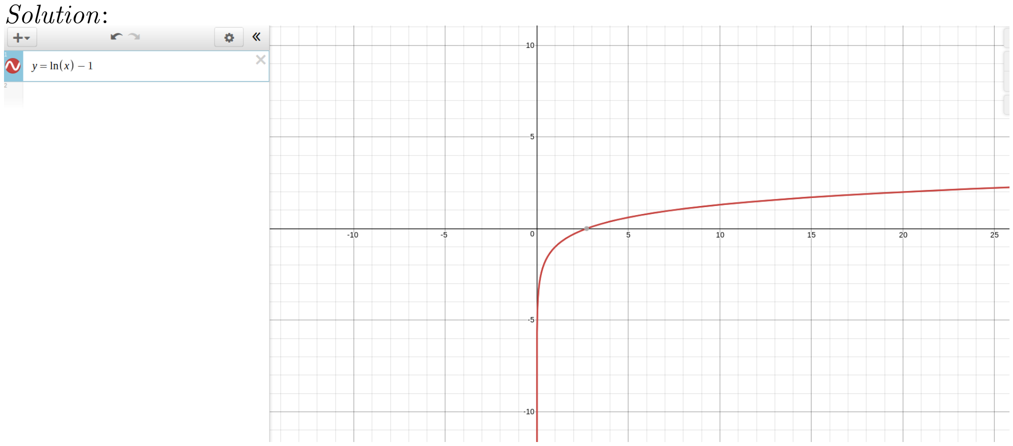
Section 1.5 #243: Sketch the graph of $f(x) = 1 - 2^{-x}$. Determine the domain, range, and horizontal asymptote.
Solution:



The domain is all real numbers (it may appear to not be, but any negative x may be plugged into this function!) and the range is $(-\infty, 1)$.
Section 1.5 #268: Sketch the graph of the logarithmic function. Determine the domain, range, and vertical asymptote: $f(x) = \ln(x) - 1$.
Solution:



The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$ (it may appear to not be, but logs in fact do tend to $+\infty$!!)
Section 1.5 #275: Use the properties of logarithms to write the expression as a sum, difference, and/or product of logarithms: $\ln\left(\frac{6}{\sqrt{e^3}}\right)$.
Solution: We may break the logarithm across the quotient to get

$$\ln\left(\frac{6}{\sqrt{e^3}}\right) = \ln(6) - \ln\left(\sqrt{e^3}\right).$$

Rewrite $\sqrt{e^3}$ as $(e^3)^{\frac{1}{2}} = e^{\frac{3}{2}}$ and use the property of logs relating to exponents to see that

$$\ln(6) - \ln(\sqrt{e^3}) = \ln(6) - \ln(e^{\frac{3}{2}}) = \ln(6) - \frac{3}{2}\ln(e).$$

Since $\ln(e) = 1$ (this is by definition!), we are finished and can write

$$\ln\left(\frac{6}{\sqrt{e^3}}\right) = \ln(6) - \frac{3}{2}.$$

Section 1.5 (like #277): Solve exactly: $e^{7x} - 15 = 0$.
Solution: To solve this, first isolate e^{7x} in the equation to get $e^{7x} = 15$. Since $\ln(x)$ is the inverse function of e^x take ln of both sides yielding

$$\cancel{\ln(e}^{\,7x}) = \ln(15)$$

yielding

$$7x = \ln(15).$$

Now divide by 7 to finally get

$$x = \frac{\ln(15)}{7}.$$

Section 1.5 (like #288): Solve exactly: $\ln(\sqrt{x+3}) = 2$.
Solution: First write $\sqrt{x+3} = (x+3)^{\frac{1}{2}}$. Using the property of logs relating to exponents, we get

$$\ln(\sqrt{x+3}) = \ln\left((x+3)^{\frac{1}{2}}\right) = \frac{1}{2}\ln(x+3).$$

This means the original equation becomes

$$\frac{1}{2}\ln(x+3) = 2.$$

Multiply by 2 to get

$$\ln(x+3) = 4.$$

To solve for x we exploit the fact that e^x is the inverse of $\ln(x)$ by plugging both sides of the equation into e^x to get

$$e^{\ln(x+3)} = e^4,$$

so

$$\cancel{e}^{\cancel{\ln}(x+3)} = e^4$$

yielding

$$x + 3 = e^4.$$

To find x subtract 3 to get

$$x = e^4 - 3.$$

Section 1.5 #300: The demand D (in millions of barrels) for oil in an oil-rich country is given by the function $D(p) = 150 \cdot (2.7)^{-0.25p}$, where p is the price (in dollars) of a barrel of oil. Find the amount of oil demanded (to the nearest million barrels) when the price is between \$15 and \$20.
Solution: The amount of oil demanded when the price is \$15 is given by

$$D(15) = 150(2.7)^{-0.25(15)} \approx 3.61 \text{ millions of barrels}$$

and the amount of oil demanded when the price is \$20 is given by

$$D(20) = 150(2.7)^{-0.25(20)} \approx 1.04 \text{ millions of barrels.}$$

Therefore the amount of oil demanded when the price is between \$15 and \$20 is between 1.04 millions of barrels and 3.61 millions of barrels.
Section 1.5 #308: The rabbit population on a game reserve doubles every 6 months. Suppose there were 120 rabbits initally.

a. Use the exponential function $P(t) = P_0 a^t$ (where t is measured in months) to determine the growth rate constant a. Round to four decimal places.
Solution: We are told that the initial population P_0 is $P_0 = 120$ – note that this means that $P(0) = 120a^0 = 120(1) = 120$ (as expected). We are told that after 6 months, the population doubles. This means that

$$\underbrace{2 \cdot 120 = 240}_{\text{given: double the initial population}} = P(6) = \underbrace{120a^6}_{\text{calculated}}$$

This gives us the equation $240 = 120a^6$. Therefore we may solve for a by first dividing by 120 to get

$$\frac{240}{120} = a^6,$$

hence

$$2 = a^6.$$

To solve for a we just take both sides to the $\frac{1}{6}$ power (this is the same as taking the 6th root) to get

$$2^{\frac{1}{6}} = \left(a^6\right)^{\frac{1}{6}} = a.$$

Thus we have growth constant $a = 2^{\frac{1}{6}} \approx 1.1224$.

b. Use the function in part a. to determine approximately how long it takes for the rabbit population to reach 3500.
Solution: We know from part a. that the function P is given by the formula $P(t) = 120(1.1224^t)$. We are being asked to solve for the time t for which $P(t) = 3500$, in other words we must solve the equation $120(1.1224)^t = 3500$. To solve it, divide both sides by 120 to get

$$1.1224^t = \frac{3500}{120}.$$

To isolate t, we *could* take the log with base 1.1224 of both sides, but that is awkward to compute. It is much easier to just plug both sides into $\ln(x)$ to get

$$\ln(1.1224^t) = \ln\left(\frac{3500}{120}\right).$$

Using the property that logs bring exponents to the front, we get

$$t\ln(1.1224) = \ln\left(\frac{3500}{120}\right).$$

Finally, divide by $\ln(1.1224)$ to get

$$t = \frac{\ln\left(\frac{3500}{120}\right)}{\ln(1.1224)} \approx 29.2114 \text{ months}$$