HW9 MATH 2501 Fall 2018

Section 4.3

#109] Find critical pts of $y = 4\sqrt{x} - x^2$

Soln: $y' = \frac{2}{\sqrt{x}} - 2x \overset{\text{set}}{=} 0$

$\frac{2}{\sqrt{x}}$ doesn't exist at $x=0$ → so that is a crit. pt.

$\frac{2}{\sqrt{x}} - 2x = 0$

↓ multiply by $\sqrt{x}$

$2 - 2x^{3/2} = 0$

↓

$x^{3/2} = 1$

↓ square both sides

$x^3 = 1$

↓

$x = \sqrt[3]{1} = 1$

critical points are at $x=0$ and $x=1$

#111] crit. pts. of $y = \ln(x-2)$

Soln: $y' = \frac{1}{x-2}$

DNE at $x=2$

$\frac{1}{x-2} = 0$

↓ multiply by $x-2$

$1 = 0$

FALSE (no soln)

only c.p. is at $x=2$

#116 $y = \sin^2(x) \longrightarrow y' = 2\sin(x)\cos(x) \overset{\text{set}}{=} 0$

↓ div by 2

$\sin(x)\cos(x) = 0$

↙ ↘

$\sin(x) = 0$ $\quad$ $\cos(x) = 0$

↓ ↓

$x = n\pi$ for any integer n $\quad$ $x = (2n+1)\frac{\pi}{2}$ for any integer n

("odd multiples of $\pi/2$")

So critical pts are

$x = n\pi$ or $x = (2n+1)\pi/2$ for $n \in \mathbb{Z}$

#118 $f(x) = x^2 + 3, \quad [-1, 4)$

Soln: $f'(x) = 2x \overset{\text{set}}{=} 0 \longrightarrow x = 0$

$x =$	$f(x) =$	
-1	4	
0	3	← abmin of 3 occurs at $x = 0$
4	19	← abmax of 19 occurs at $x = 4$

#120 $f(x) = (x - x^2)^2, \quad [0, 1]$

Soln: $f'(x) = \boxed{2(x-x^2)(1-2x) \overset{\text{set}}{=} 0}$

↓ div by 2

$(x-x^2)(1-2x) = 0$

↙ ↘

$x - x^2 = 0$ $\quad$ $1 - 2x = 0$

↓ ↓

$x = 0$ or $x = 1$ $\quad$ $x = 1/2$

x=	f(x)=
0	0
1/2	1/16
1	0

Abmin of 0 occurs at both $x=0$ and $x=1$

Abmax of $\frac{1}{16}$ occurs at $x=1/2$.

(3)

#122 $y=\sqrt{9-x}$, $[1,9]$

Soln: $y' = \frac{-1}{2\sqrt{9-x}}$ → any c.p. is at $x=9$ where y' DNE

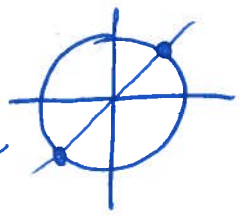
x=	y=
1	$\sqrt{8}$ → abmax of $\sqrt{8}$ occurs at $x=1$
9	0 → abmin of 0 occurs at $x=9$

#127 $y=\sin(x)+\cos(x)$, $[0,2\pi]$

Soln: $y' = \cos(x)-\sin(x) \overset{\text{set}}{=} 0$

$\cos(x)=\sin(x)$

$x=\frac{\pi}{4}, \frac{5\pi}{4}$



x=	y=
0	1
$\pi/4$	$\sqrt{2}$ → Abmax of $\sqrt{2}$ occurs at $x=\pi/4$
$5\pi/4$	$-\sqrt{2}$ → Abmin of $-\sqrt{2}$ occurs at $x=5\pi/4$
2π	1

#140 $C = x^2 - 1200x + 36400$

$C' = 2x - 1200 \overset{\text{set}}{=} 0 \longrightarrow x = 600$

Section 4.4

#161 $f(x) = x^3$

$f'(x) = 3x^2$

$f(2) - f(0) = f'(c)(2-0)$

$8 - 0 = f'(c)(2)$

$f'(c) = 4$

Solve

$4 = f'(c) = 3c^2$

4: from $f(2) - f(0) = f'(c)(2-0)$; $3c^2$: from calculation

$4 = 3c^2 \longrightarrow c^2 = \frac{4}{3}$

$c = \pm \frac{2}{\sqrt{3}}$

#190 10:17AM → pass 1st car; 10:53AM → pass 2nd car → 36 minutes between these events

traveled 39 miles in this time period

Average speed $= \frac{39}{36} \approx 1.0833 \frac{\text{miles}}{\text{minute}} = \left(1.0833 \frac{\text{miles}}{\text{min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)$

$= 65 \frac{\text{miles}}{\text{hr}}$

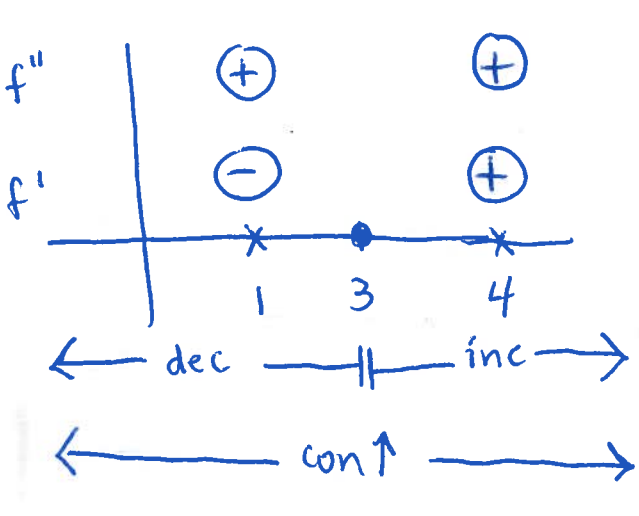
Yes → MVT says that there had to be a moment during your trip where you went 65 $\frac{\text{mi}}{\text{hr}}$, which is above the posted speed limit of 60 $\frac{\text{mi}}{\text{hr}}$.

Section 4.5

#224 $f(x)=x^2-6x$

$f'(x)=2x-6 \overset{\text{set}}{=} 0 \rightarrow x=3$

$f''(x)=2 \overset{\text{set}}{=} 0 \rightarrow$ NO SOLN

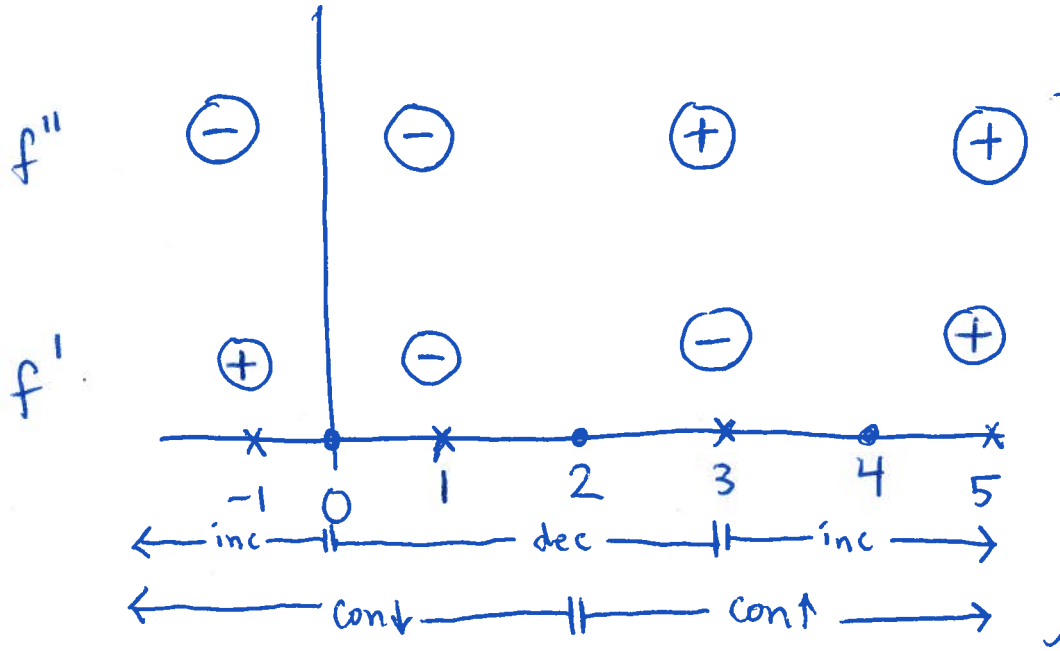


- local min at $x=3$
- no inflection pt

#225 $f(x)=x^3-6x^2$

$f'(x)=3x^2-12x \overset{\text{set}}{=} 0 \rightarrow x(3x-12)=0$ → $x=0$ OR $3x-12=0 \rightarrow x=4$

$f''(x)=6x-12 \overset{\text{set}}{=} 0 \rightarrow x=2$



- local max at $x=-1$
- local min at $x=4$
- inflection pt at $x=2$