HW7 MATH 2501 Fall 2018

Section 3.9

#340 $\frac{d}{dx}\ln(4x^3+x) = \left(\frac{1}{4x^3+x}\right)\underbrace{\frac{d}{dx}(4x^3+x)}_{\text{chain rule}} = \frac{12x^2+1}{4x^3+x}$

#342 $\frac{d}{dx}x^2\ln(9x) = \frac{d}{dx}(x^2)\ln(9x) + x^2\frac{d}{dx}\ln(9x)$

product Rule

$= 2x\ln(9x) + x^2 \cdot \frac{1}{9x}\underbrace{\frac{d}{dx}[9x]}_{\text{Chain Rule}}$

$= 2x\ln(9x) + \frac{9x}{9}$

$= 2x\ln(9x) + x$

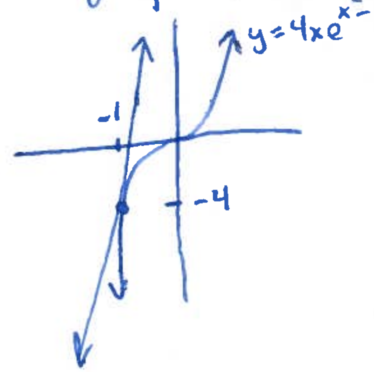
product rule and chain rule

#354 $f'(x) = 4e^{x^2-1} + 4x(e^{x^2-1})(2x) = (e^{x^2-1})(4+8x^2)$. The function f at -1 yields $f(-1) = -4$

Therefore, slope of tangent line is $f'(-1) = e^0(4+8) = 12$

Therefore, equation of tangent line is eqt of line thru $(-1,-4)$ w/ slope 12:

$y+4 = 12(x+1)$



#360 a) $M = M_0 e^{kt}$

mass (amount of substance)

initial mass

$M = 9e^{kt} \xrightarrow{\text{half-life}} 4.5 = 9e^{12k} \rightarrow \frac{1}{2} = e^{k(12)} \rightarrow \ln(\frac{1}{2}) = k(12)$

$\rightarrow k = \frac{1}{12}\ln(\frac{1}{2}) \approx -0.0577$

Therefore, amount of substance at time t, measured in hours is

$M = 9e^{\frac{1}{12}\ln(\frac{1}{2})\,t}$

b) $\frac{d}{dt}M = \frac{d}{dt}\left(9e^{\frac{1}{12}\ln(\frac{1}{2})t}\right) = \frac{9}{12}\ln(\frac{1}{2})\,e^{\frac{1}{12}\ln(\frac{1}{2})t}$ $\frac{\text{grams}}{\text{hour}}$

c) $\left.\frac{dM}{dt}\right|_{t=4} = \frac{9}{12}\ln(\frac{1}{2})e^{\frac{1}{12}\ln(\frac{1}{2})(4)} \approx -0.4126$ $\frac{\text{grams}}{\text{hour}}$

Section 4.1

#4 Given:

$$\boxed{\begin{array}{ll} \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} & \\ \frac{dR_1}{dt} = 0.5 \frac{\Omega}{min} & \frac{dR_2}{dt} = -1.1 \frac{\Omega}{min} \\ R_1 = 20\ \Omega & R_2 = 50\ \Omega \end{array}}$$

Seek: $\frac{dR}{dt}$

Soln: Compute $\frac{d}{dt}$ of both sides of $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$:

$$\frac{d}{dt}(R^{-1}) = \frac{d}{dt}(R_1^{-1} + R_2^{-1})$$

$$-R^{-2}\frac{dR}{dt} = -R_1^{-2}\frac{dR_1}{dt} - R_2^{-2}\frac{dR_2}{dt}$$

$\downarrow$ negative exponents AND multiply by -1

$$\frac{1}{R^2}\frac{dR}{dt} = \frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}$$

Multiply by R^2 to get

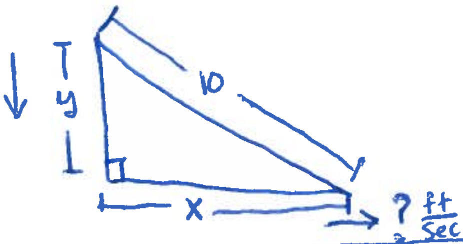
$$\frac{dR}{dt} = \frac{R^2}{R_1^2}\frac{dR_1}{dt} + \frac{R^2}{R_2^2}\frac{dR_2}{dt}$$

Note that when $R_1 = 20$ and $R_2 = 50$, $R = \dfrac{1}{\frac{1}{20} + \frac{1}{50}} = \dfrac{100}{7}$

Therefore, compute

$$\left.\frac{dR}{dt}\right|_{\substack{R_1=20,\, R_2=50,\, R=\frac{100}{7} \\ \frac{dR_1}{dt}=0.5,\, \frac{dR_2}{dt}=-1.1}} = \frac{\left(\frac{100}{7}\right)^2}{20^2}(0.5) + \frac{\left(\frac{100}{7}\right)^2}{50^2}(-1.1) \approx 0.1653 \frac{\Omega}{min}$$

#5



Given: $\frac{dy}{dt} = -2 \frac{ft}{sec}$

$x = 5$, Pythagorean theorem: $x^2 + y^2 = 100$

Seek: $\frac{dx}{dt}$

Soln: Take $\frac{d}{dt}$ of Pythagorean theorem to get

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-2y}{2x} \frac{dy}{dt}$$

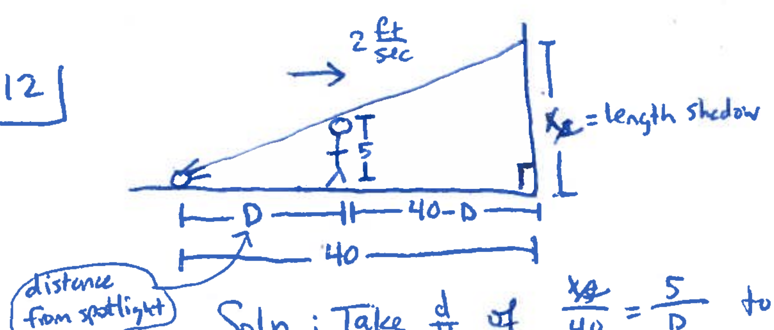
$$\frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$$

Note, at $x = 5$, $y = \sqrt{75}$, from $x^2 + y^2 = 100$.

Therefore,

$$\left.\frac{dx}{dt}\right|_{\substack{\frac{dy}{dt} = -2 \\ x = 5 \\ y = \sqrt{75}}} = -\frac{\sqrt{75}}{5}(-2) = \frac{2\sqrt{75}}{5} \frac{ft}{sec} \approx 3.46 \frac{ft}{sec}$$

#12



Given: $\frac{dD}{dt} = 2 \frac{ft}{sec}$

$\frac{x_s}{40} = \frac{5}{D}$ $\quad 40 - D = 10 \rightarrow D = 30$

Seek: $\frac{dx_s}{dt}$

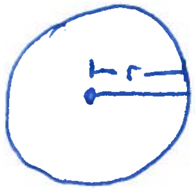
Soln: Take $\frac{d}{dt}$ of $\frac{x_s}{40} = \frac{5}{D}$ to get $\frac{1}{40} \frac{dx_s}{dt} = -\frac{5}{D^2} \frac{dD}{dt}$

$$\Rightarrow \frac{dx_s}{dt} = \frac{-200}{D^2} \frac{dD}{dt}$$

Therefore,

$$\left.\frac{dx_s}{dt}\right|_{\substack{\frac{dD}{dt} = 2 \\ D = 30}} = \left(\frac{-200}{30^2}\right)(2) \approx -0.444 \frac{ft}{sec}$$

#18



Given: $\frac{dr}{dt} = 2\,\frac{m}{sec}$, $r = 5$

$A = \pi r^2$

Seek: $\frac{dA}{dt}$

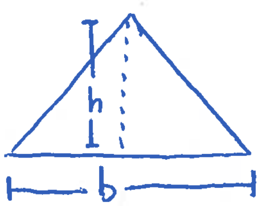
Soln: Take $\frac{d}{dt}$ of $A = \pi r^2$ to get

$$\frac{dA}{dt} = 2\pi r \,\frac{dr}{dt}$$

Therefore,

$$\left.\frac{dA}{dt}\right|_{\frac{dr}{dt}=2,\, r=5} = 2\pi(5)(2) = 20\pi \approx 62.83\,\frac{ft^2}{sec}$$

#22



Given: $\frac{db}{dt} = -1\,\frac{cm}{min}$

$h = 22$ $\quad \frac{dh}{dt} = 5\,\frac{cm}{min}$

$b = 10$

$A = \frac{1}{2} b h$

Seek: $\frac{dA}{dt}$

Soln: Take $\frac{d}{dt}$ of $A = \frac{1}{2} b h$ to get

$$\frac{dA}{dt} = \frac{1}{2} h \frac{db}{dt} + \frac{1}{2} b \frac{dh}{dt}$$

Therefore,

$$\left.\frac{dA}{dt}\right|_{\substack{\frac{db}{dt}=-1,\ \frac{dh}{dt}=5 \\ h=22,\ b=10}} = \frac{1}{2}(22)(-1) + \frac{1}{2}(10)(5)$$

$$= -11 + 25$$

$$= 14\,\frac{cm^2}{min}$$