HW6 MATH 2501 Fall 2018

Section 3.6

#228 $y = (3x^2+3x-1)^4$

Soln: $\frac{d}{dx} y = \frac{d}{dx}(3x^2+3x-1)^4$

$u = 3x^2+3x-1$, $\frac{du}{dx} = 6x+3$

$$= \frac{du}{dx}\frac{d}{du} u^4$$
$$= (6x+3)\,4u^3$$
$$= 4(6x+3)(3x^2+3x-1)^3$$

#232 $y = \frac{1}{\sin^2(x)}$

Soln: First rewrite

$$\frac{1}{\sin^2 x} = (\sin(x))^{-2}$$

Now compute

$$\frac{d}{dx} y = \frac{d}{dx}(\sin(x))^{-2}$$

$u = \sin x$, $\frac{du}{dx} = \cos(x)$

$$= \frac{du}{dx}\frac{d}{du} u^{-2}$$
$$= \cos(x)(-2u^{-3})$$
$$= -2\cos(x)(\sin(x))^{-3}$$
$$= \frac{-2\cos(x)}{\sin^3(x)}$$

#238 $y = [f(x)]^3$ and suppose that $f'(1) = 4$ and ($\frac{dy}{dx} = 10$ when $x = 1$)

Find $f(1)$.

Soln: Calculate first

$$\frac{d}{dx} y = \frac{d}{dx}[f(x)]^3$$

$u = f(x)$, $\frac{du}{dx} = f'(x)$

$$= \frac{du}{dx}\frac{d}{du} u^3$$
$$= f'(x)\cdot 3u^2$$
$$= 3f'(x)[f(x)]^2$$

Now at $x = 1$ we get

$$\underbrace{\left.\frac{dy}{dx}\right|_{x=1}}_{\text{given} \to\, =10} = 3\underbrace{f'(1)}_{\text{given} \to\, =4}[f(1)]^2$$

$$10 = 12 f(1)^2 \Rightarrow f(1) = \pm\sqrt{\frac{10}{12}}$$

(two possible solns)

#242 Find eqt of tangent line to $y = (3x + \frac{1}{x})^2$ at $(x_0, y_0) = (1, 16)$. Graph.

Soln: First compute

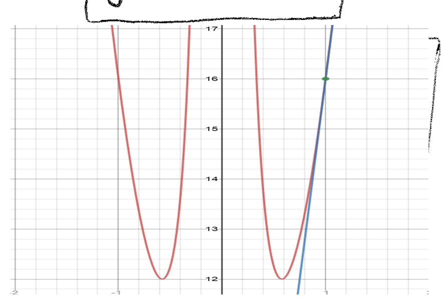
$$\frac{dy}{dx} = 2(3x + \tfrac{1}{x})(3 - \tfrac{1}{x^2})$$

Therefore slope of tangent line can be found by plugging in $x = 1$:

$$\left.\frac{dy}{dx}\right|_{x=1} = 2(3+1)(3-1) = 2(4)(2) = 16$$

Thus taking $(x_0, y_0) = (1, 16)$ in point-slope form of the equation of the line $(y - y_0 = m(x - x_0))$ to get

$$\boxed{y - 16 = 16(x-1)}$$



#256| $A = \pi r^2$, r = radius of circle

a) Suppose $r = 2 - \dfrac{100}{(t+7)^2}$ where t is measured in seconds.

Use chain rule to find rate that area is expanding.

b) Use part a) to find rate area is expanding at $t = 4$ sec

Soln: a) Compute the rate that area is expanding:

$$\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right) = \pi \frac{d}{dt} r^2$$

$$= \pi \left(\frac{dr}{dt}\right)\underbrace{\left(\frac{d}{dr} r^2\right)}_{=2r}$$

$$\frac{dr}{dt} = \frac{d}{dt}\left[2 - \frac{100}{(t+7)^2}\right]$$

rewrite

$$= \frac{d}{dt}\left[2 - 100(t+7)^{-2}\right]$$

$$= 0 - 100(-2)(t+7)^{-3}$$

$$= 200(t+7)^{-3}$$

$$= \pi\left(\frac{200}{(t+7)^3}\right)(2r)$$

$$= \pi\left(\frac{200}{(t+7)^3}\right)(2)\left(2 - \frac{100}{(t+7)^2}\right)$$

$$= \frac{400\pi}{(t+7)^3}\left(2 - \frac{100}{(t+7)^2}\right) \quad \frac{(\text{radius units})^2}{\text{Second}}$$

we weren't told

b) Rate expanding at $t = 4$:

$$\left.\frac{dA}{dt}\right|_{t=4} = \frac{400\pi}{11^3}\left(2 - \frac{100}{11^2}\right) \approx 1.107987 \; \frac{(\text{radius units})^2}{\text{second}}$$

#258| $T(x) = 94 - 10\cos\left(\frac{\pi}{12}(x-2)\right)$ $^{\circ}F$, where x is hours after midnight

Find rate temperature changing at 4PM

Soln: First compute

$$T'(x) = 0 + 10\sin\left(\frac{\pi}{12}(x-2)\right)\left(\frac{\pi}{12}\right) = \frac{10\pi}{12}\sin\left(\frac{\pi}{12}(x-2)\right) \quad \frac{{}^{o}F}{\text{hours after midnight}}$$

Since 4PM occurs 16 hours after midnight, we find the desired rate is

$$T'(16) = \frac{10\pi}{12}\sin\left(\frac{\pi}{12}(16-2)\right) = \frac{10\pi}{12}\sin\left(\frac{14\pi}{12}\right) \approx -1.3089 \quad \frac{{}^{o}F}{\text{hour after midnight}}$$

Section 3.8

#300 $x^2 - y^2 = 4$ (Find $\frac{dy}{dx}$)

Soln: Compute

$$\frac{d}{dx}(x^2 - y^2) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}x^2 - \frac{dy}{dx}\frac{d}{dy}(y^2) = 0$$

$$2x - \frac{dy}{dx}(2y) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{2y} = \frac{x}{y}$$

#302 $x^2 y = y - 7$

Soln: Using product rule on left,

$$2xy + x^2\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow x^2\frac{dy}{dx} - \frac{dy}{dx} = -2xy$$

$$(x^2 - 1)\frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 - 1}$$

#304 $xy - \cos(xy) = 1$

Soln: Using product and chain rule,

$$\left(y + x\frac{dy}{dx}\right) + \sin(xy)\underbrace{\frac{d}{dx}(xy)}_{= y + x\frac{dy}{dx}} = 0$$

$$\left(y + x\frac{dy}{dx}\right) + \sin(xy)\left(y + x\frac{dy}{dx}\right) = 0$$

$$x\frac{dy}{dx} + x\sin(xy)\frac{dy}{dx} = -y - y\sin(xy)$$

$$\left(\frac{dy}{dx}\right)(x + x\sin(xy)) = -(y + y\sin(xy))$$

$$\frac{dy}{dx} = \frac{-(y + y\sin(xy))}{x + x\sin(xy)}$$

#314 Find eqt of tangent line to

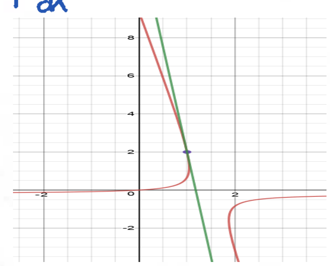
$$\frac{x}{y} + 5x - 7 = -\frac{3}{4}y \quad \text{at } (x_0, y_0) = (1, 2)$$

Soln: First take $\frac{d}{dx}$ of both sides to get

$$\underbrace{\frac{(y)(1) - x\frac{dy}{dx}}{y^2}}_{\text{quotient rule on } \frac{x}{y}} + 5 - 0 = -\frac{3}{4}\frac{dy}{dx}$$

$\left(= \frac{1}{y}\right)$

$$\Rightarrow \frac{y}{y^2} - \frac{x}{y^2}\frac{dy}{dx} + 5 = -\frac{3}{4}\frac{dy}{dx}$$



$$\Rightarrow \left(\frac{3}{4} - \frac{x}{y^2}\right)\frac{dy}{dx} = -5 - \frac{1}{y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(5 + \frac{1}{y})}{\left(\frac{3}{4} - \frac{x}{y^2}\right)} \Rightarrow \left.\frac{dy}{dx}\right|_{\substack{x=1 \\ y=2}} = \frac{-(5 + \frac{1}{2})}{\left(\frac{3}{4} - \frac{1}{4}\right)} = \frac{-11/2}{2/4}$$

$$= \frac{-11}{2}\left(\frac{4}{2}\right)$$

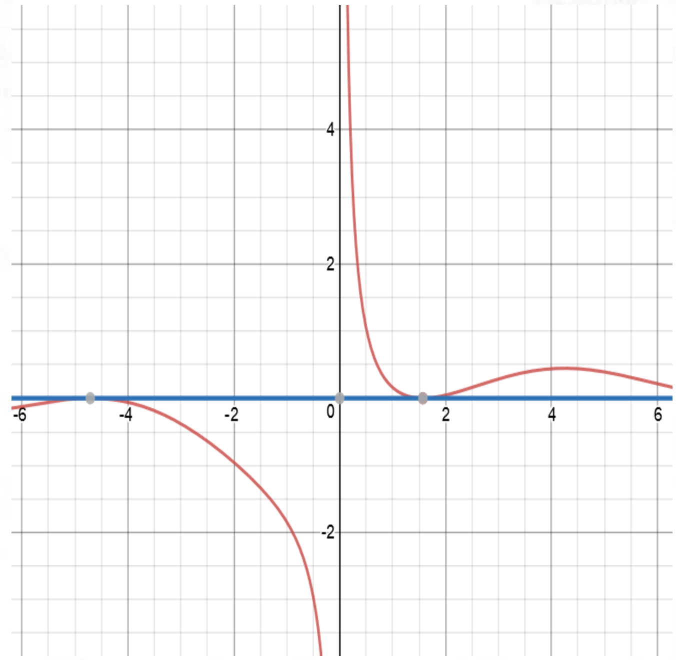
$$= -11$$

Therefore eqt of tangent line is

$$y - 2 = -11(x - 1)$$

Find eq^t of tangent line to

#315 $xy + \sin(x) = 1$ at $\left(\frac{\pi}{2}, 0\right)$



Soln: Take $\frac{d}{dx}$ to get

$\theta = \pi/2$
$(0,1)$

$$y + x\frac{dy}{dx} + \cos(x) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\cos(x) - y}{x}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\substack{x=\pi/2 \\ y=0}} = \frac{-\cos(\pi/2) - 0}{\pi/2} = \frac{0-0}{\pi/2} = 0$$

Therefore equation of tangent line is

$$y - 0 = 0(x - \pi/2)$$

$$\Downarrow$$

$$y = 0$$

#316 $2x^3 + 2y^3 - 9xy = 0$ at $(2,1)$

Soln: (a) Take $\frac{d}{dx}$ to get

$$6x^2 + 6y^2\frac{dy}{dx} - 9\left(y + x\frac{dy}{dx}\right) = 0$$

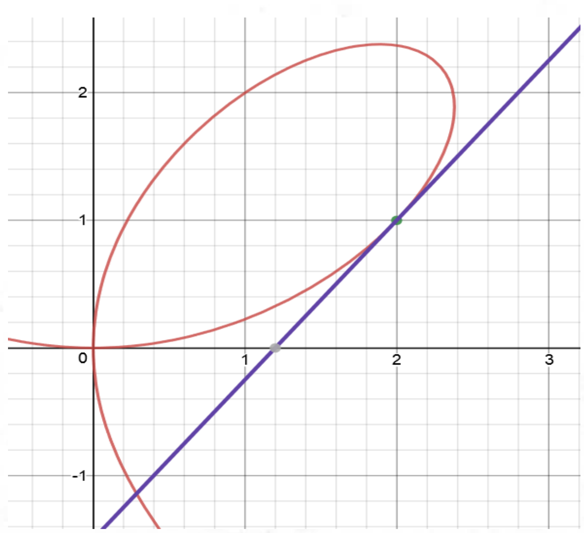
$$\Rightarrow (6y^2 - 9x)\frac{dy}{dx} = 9y - 6x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{9y - 6x^2}{6y^2 - 9x}$$

Calculate slope at

$$\left.\frac{dy}{dx}\right|_{\substack{x=2 \\ y=1}} = \frac{9(1) - 6(2^2)}{6(1^2) - 9(2)} = \frac{9-24}{6-18} = \frac{-15}{-12} = \frac{15}{12}$$

$$\Rightarrow y - 1 = \frac{15}{12}(x - 2)$$



Section 3.4

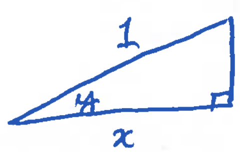
#329 $x = \cos(y)$

Soln: Take $\frac{d}{dx}$ to get

$$1 = -\sin(y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sin(y)}$$

Now, since $\cos(y) = x$, we draw a Δ to get



$$? \Rightarrow x^2 + ?^2 = 1$$
$$\Rightarrow ? = \sqrt{1-x^2}$$

Therefore, $\sin(y) = \sqrt{1-x^2}$,

thus $\frac{dy}{dx} = -\frac{1}{\sin(y)} = -\frac{1}{\sqrt{1-x^2}}$

(note: this establishes that $\frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$

#332 $f(x) = \frac{e^{-x}}{x}$

Soln:

$$f'(x) = \frac{x(-e^{-x}) - e^{-x}(1)}{x^2}$$
$$= e^{-x}\left(\frac{-x-1}{x^2}\right)$$
$$= -e^{-x}\left(\frac{x-1}{x^2}\right)$$

#334 $f(x) = \sqrt{e^{2x} + 2x}$

Soln: First rewrite

$$f(x) = (e^{2x} + 2x)^{1/2}$$

Now compute

$$f'(x) = \frac{1}{2}(e^{2x} + 2x)^{-1/2} \underbrace{\frac{d}{dx}(e^{2x} + 2x)}_{= 2e^{2x} + 2}$$
$$= \frac{2e^{2x} + 2}{2\sqrt{e^{2x} + 2x}} = \frac{e^{2x} + 1}{\sqrt{e^{2x} + 2x}}$$

#336 $f(x) = \frac{10^x}{\ln(10)}$

Soln: First rewrite $10^x = e^{\ln(10^x)} = e^{x\ln(10)}$

so $f(x) = \frac{e^{x\ln(10)}}{\ln(10)}$. Therefore,

$$f'(x) = \frac{d}{dx}\left(\frac{e^{x\ln(10)}}{\ln(10)}\right) = \frac{1}{\ln(10)}\frac{d}{dx}\left(e^{x\ln(10)}\right)$$
$$= \frac{1}{\cancel{\ln(10)}}\left[e^{x\ln(10)}\right](\cancel{\ln(10)})$$
$$= e^{x\ln(10)} = 10^x$$

#338 $f(x) = 3^{\sin(3x)}$

Soln: Rewrite

$$f(x) = 3^{\sin(3x)} = e^{\ln(3^{\sin(3x)})}$$
$$= e^{\sin(3x)\ln(3)}$$

Now compute

$$f'(x) = \underbrace{e^{\sin(3x)\ln(3)}}_{= 3^{\sin(3x)}} \frac{d}{dx}(\ln(3)\sin(3x))$$
$$= 3^{\sin(3x)} \cdot \ln(3) \cdot 3\cos(3x)$$
$$= 3^{1+\sin(3x)}\ln(3)\cos(3x)$$