HW5 MATH 2501 Fall 2018

Section 3.3

rewrite $\frac{1}{x} = x^{-1}$

#110 $f(x) = x^4 + \frac{2}{x} = x^4 + 2x^{-1}$

$$f'(x) = \frac{d}{dx}[x^4] + \frac{d}{dx}\left[\frac{2}{x}\right]$$

quotient RULE

$$= 4x^3 + \frac{x\frac{d}{dx}[2] - 2\frac{d}{dx}[x]}{x^2}$$

$$= 4x^3 + \frac{(x)(0) - 2(1)}{x^2}$$

$$= 4x^3 - \frac{2}{x^2}$$

OR

$$f'(x) = \frac{d}{dx}[x^4] + \frac{d}{dx}[2x^{-1}]$$

$$= 4x^3 + 2(-1)x^{-1-1}$$

$$= 4x^3 - 2x^{-2}$$

optional $= 4x^3 - \frac{2}{x^2}$

#112 $f(x) = (x+2)(2x^2-3)$

$$f'(x) = \frac{d}{dx}\left[(x+2)(2x^2-3)\right]$$

product RULE

$$= \underbrace{\left(\frac{d}{dx}[x+2]\right)}_{=1}(2x^2-3) + (x+2)\underbrace{\left(\frac{d}{dx}[2x^2-3]\right)}_{=4x}$$

$$= 1(2x^2-3) + (x+2)(4x)$$

$$= (2x^2-3) + (4x^2+8x)$$

$$= 6x^2 + 8x - 3$$

#116 $f(x) = \frac{x^2+4}{x^2-4} \Rightarrow f'(x) = \frac{d}{dx}\left[\frac{x^2+4}{x^2-4}\right]$

$\boxed{\text{quotient RULE}}$

$$= \frac{(x^2-4)\frac{d}{dx}[x^2+4] - (x^2+4)\frac{d}{dx}[x^2-4]}{(x^2-4)^2}$$

$$= \frac{(x^2-4)(2x) - (x^2+4)(2x)}{(x^2-4)^2} = \frac{(\cancel{2x^3}-8x) - (\cancel{2x^3}+8x)}{(x^2-4)^2} = \frac{-16x}{(x^2-4)^2}$$

#118: Find equation of tangent line to $y=3x^2+4x+1$ at $(x_0,y_0)=(0,1)$

Soln: Line obeys $y-y_0=m(x-x_0)$

m = slope of tangent line $= y'(x_0)$

Calculate

$$y'(x)=\frac{dy}{dx}=\frac{d}{dx}\left[3x^2+4x+1\right]=6x+4$$

Therefore

$$m=y'(x_0)=6(x_0)+4$$

$(x_0,y_0)=(0,1)$

$$=6(0)+4$$

$$=4$$

Therefore equation of tangent line is

$$y-1=4(x-0)$$

(add 1 both sides)

$$y=4x+1$$

#138 Find equation of tangent line to graph of

$$y = x^2 + \frac{4}{x} - 10 \quad \text{at} \quad (x_0, y_0) = (8, ?).$$

$$\begin{array}{r} 54 \\ \times 2 \\ \hline 108 \end{array}$$

Soln: First find ? by plugging $x_0 = 8$ into the equation to get

$$y\Big|_{x=8} = 8^2 + \frac{4}{8} - 10 = 64 + \frac{1}{2} - 10$$

$$= 54 + \frac{1}{2}$$

$$= \frac{108}{2} + \frac{1}{2}$$

$$= \frac{109}{2}$$

Therefore, $(x_0, y_0) = (8, \frac{109}{2})$.

Now find slope of tangent line:

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^2 + 4/x\right] = \frac{d}{dx}\left[x^2 + 4x^{-1}\right]$$

algebra: $\frac{1}{x} = x^{-1}$

$$= 2x - 4x^{-2} = 2x - \frac{4}{x^2}$$

So slope at (x_0, y_0) given by plugging $x_0 = 8$ in for x to get

$$m = \frac{dy}{dx}\Big|_{x=8} = 2(8) - \frac{4}{8^2} = 16 - \frac{4}{64} = 16 - \frac{1}{16}$$

$$= \frac{256}{16} - \frac{1}{16}$$

$$= \frac{255}{16}$$

Therefore, equation of tangent line thru (x_0, y_0) is

$$y - \frac{109}{2} = \frac{255}{16}(x - 8).$$

#146

$$P(t) = \frac{8t+3}{0.2t^2+1} \text{ millions of flounder}$$

t measured in years

a) Find initial flounder population.

b) Determine $P'(10)$ and interpret.

Soln:

(a) Initial population occurs at $t=0$:

$$P(0) = \frac{8(0)+3}{(0.2)(0^2)+1} = \frac{0+3}{0+1} = \frac{3}{1} = 3 \text{ million flounder}$$

(b) $$P'(t) = \frac{d}{dt}\left[\frac{8t+3}{0.2t^2+1}\right]$$

$$\overset{\text{quotient rule}}{=} \frac{(0.2t^2+1)(8)+(8t+3)(0.4t)}{(0.2t^2+1)^2},$$

so

$$P'(10) = \frac{[(0.2)\overbrace{(10^2)}^{=100}+1](8)+\overbrace{[8(10)+3]}^{=80}(0.4)(100)}{[(0.2)\underbrace{(10^2)}_{=100}+1]^2}$$

$$= \frac{[20+1](8)+[83](4)}{[20+1]^2}$$

$$= \frac{168-332}{441} = \underbrace{\frac{-164}{441}}_{\approx -0.3718} \frac{\text{millions of flounder}}{\text{year}}$$

$$\begin{array}{r} 21 \\ \times 8 \\ \hline 168 \end{array} \quad \begin{array}{r} 83 \\ \times 4 \\ \hline 332 \end{array} \quad \begin{array}{r} 21 \\ \times 21 \\ \hline 21 \\ +420 \\ \hline 441 \end{array} \quad \begin{array}{r} 332 \\ -168 \\ \hline 164 \end{array}$$

This means at the 10^{th} year, our model predicts that the flounder population will be decreasing at a rate of 0.3718 $\frac{\text{millions of flounder}}{\text{year}}$.

the minus sign

Section 3.4 #164 $P(t) = -\frac{1}{3}t^3 + 64t + 3000$

a) $P'(t) = -t^2 + 64$ $\frac{\text{thousands of people}}{\text{year}}$

b) $P'(1) = -1^2 + 64 = 63$

$P'(2) = -2^2 + 64 = 60$

$P'(3) = -3^2 + 64 = 55$

$P'(4) = -4^2 + 64 = 48$

The rate of change of the town's population remains positive, but at a decreasing rate.

c) $P''(t) = -2t$ $\frac{\text{thousands of people}}{\text{year}^2}$

$P''(1) = -2$

$P''(2) = -4$

$P''(3) = -6$

$P''(4) = -8$

This tells us as years go on, the rate that P' is decreasing at is increasing.

Section 3.5 #176 $y = 3\csc(x) + \frac{5}{x}$

$\downarrow$ $\csc(x) = \frac{1}{\sin x}$; $\frac{1}{x} = x^{-1}$

$$\frac{dy}{dx} = 3\frac{d}{dx}\left(\frac{1}{\sin x}\right) + 5\frac{d}{dx}(x^{-1})$$

$$= 3\left[\frac{\sin(x)\cdot 0 - 1\cdot\cos(x)}{\sin^2(x)}\right] - 5x^{-2}$$

using $\cot(x) \overset{\text{def}}{=} \frac{\cos(x)}{\sin(x)}$, $\csc(x) \overset{\text{def}}{=} \frac{1}{\sin(x)}$, and $x^{-2} \overset{\text{def}}{=} \frac{1}{x^2}$

$$= -3\cot(x)\csc(x) - \frac{5}{x^2}$$

#180 $y = \sin(x)\tan(x)$

$\Downarrow$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(x)\tan(x)\right]$$

product rule
$$= \frac{d}{dx}[\sin(x)]\tan(x) + \sin(x)\underbrace{\frac{d}{dx}[\tan(x)]}$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} \overset{Q.R.}{=} \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2 x} = \sec^2(x)$$

$$= \cos(x)\tan(x) + \sin(x)\sec^2(x)$$
$$= \sin(x) + \sin(x)\sec^2(x)$$

#200 $s(t) = -6\cos(t)$

$\Downarrow$

"rate oscillating at $t=5$" means to find $s'(t)$ and plug in $t=5$:

$$s'(t) = \frac{d}{dt}[-6\cos(t)]$$
$$= -6\frac{d}{dt}[\cos(t)]$$
$$= -6(-\sin(t))$$
$$= 6\sin(t)\ \frac{\text{inches}}{\text{sec}}$$

So, rate oscillating at $t=5$ is
$$s'(5) = -6\sin(5) \approx 5.754\ \frac{\text{inches}}{\text{sec}}$$

#208
$$\frac{d}{dx}[\cos(x)] \overset{\text{def}}{=} \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$
$$= \lim_{h\to 0}\frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$
$$= \lim_{h\to 0}\ \cos(x)\underbrace{\left[\frac{\cos(h)-1}{h}\right]}_{0 \text{ as } h\to 0} - \sin(x)\underbrace{\left[\frac{\sin(h)}{h}\right]}_{1 \text{ as } h\to 0}$$

$$= 0 - \sin(x)$$
$$= -\sin(x)$$

Section 3.6 #214

$y = 3u - 6, \quad u = 2x^2$

$\frac{dy}{du} = 3 \qquad \frac{du}{dx} = 4x$

$$\frac{dy}{dx} = \frac{du}{dx}\frac{dy}{du}$$

$$= (4x)(3)$$

$$= 12x$$

#216

$y = \sin(u), \quad u = 5x - 1$

$\frac{dy}{du} = \cos(u) \qquad \frac{du}{dx} = 5$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \cos(u)\cdot 5$$

$$= 5\cos(5x-1)$$

#220

$y = (3x-2)^6$

Here let $u = 3x - 2$, so $\frac{du}{dx} = 3$. Thus,

$$\frac{d}{dx}\left[(3x-2)^6\right] = \frac{d}{dx}u^6$$

$$= \frac{du}{dx}\frac{d}{du}u^6$$

$$= 3(6u^5)$$

$$= 18(3x-2)^5$$

$$\frac{d}{du}\tan u = \frac{d}{du}\frac{\sin u}{\cos u}$$

$$= \frac{\cos^2 u + \sin^2 u}{\cos^2 u}$$

$$= \sec^2 u$$

#224

$y = \tan(\sec(x))$

Let $u = \sec(x)$ so $\frac{du}{dx} = \frac{d}{dx}\frac{1}{\cos x} = \frac{\cos(x)(0) - 1(-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \tan(x)\sec(x)$

Thus, $\frac{dy}{dx} = \frac{d}{dx}\tan(u) = \frac{du}{dx}\frac{d}{du}\tan(u) = \tan(x)\sec(x)\sec^2(u)$

$$= \tan(x)\sec(x)\sec^2(\sec(x))$$