MATH 2501 Fall 2018 HW3

$\boxed{\S 2.4 \ \#134}$ $g(t) = \frac{1}{t} + 1$

Problem occurs when denominator equals zero, i.e. at $t=0$ — infinite discontinuity

$\boxed{\#137}$ $H(x) = \tan(2x)$

The function $\tan(\theta)$ has ~~problems~~ asymptotes whenever

$$\begin{cases} \theta = \frac{\pi}{2} + 2\pi n, & n = \dots, -1, 0, 1, \dots \\ \theta = -\frac{\pi}{2} + 2\pi n, & n = \dots, -1, 0, 1, \dots \end{cases}$$

Therefore $H(x)$ has asymptotes at

$$\begin{cases} 2x = \frac{\pi}{2} + 2\pi n \\ 2x = -\frac{\pi}{2} + 2\pi n, \end{cases}$$

or in other ~~words~~ words, at

$$\begin{cases} x = \frac{\pi}{4} + \pi n \\ x = -\frac{\pi}{4} + \pi n \end{cases}$$

$\boxed{\#148}$ Recall "f is continuous at $x=a$" means $\begin{cases} \text{(i) } f(a) \text{ exists} \\ \text{(ii) } \lim_{x \to a} f(x) \text{ exists} \\ \text{(iii) } \lim_{x \to a} f(x) = f(a) \end{cases}$

So in this problem, ~~compute~~ the issue could occur at $x=4$, so compute

(i) $f(4) = 4+3 = 7$

(ii) To see limit, compute left- and right-limits

$\lim_{x \to 4^-} f(x) = e^{4k}$ and $\lim_{x \to 4^+} f(x) = 7$

For limit to exist we must have $e^{4k} = \lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = 7$

Thus, $e^{4k} = 7$, hence $4k = \ln(7) \Rightarrow k = \frac{\ln(7)}{4}$.

With this choice, we get continuity at $x=4$, as desired.

#152 a) $\cos(t) = t^3$

b) Look at $f(t) = \cos(t) - t^3$. This fnct is continuous.

Consider interval $[-1, 1]$

$f(-1) \approx 1.54$

$f(1) \approx -0.459$

So, by intermediate value theorem, there is a value c such that $-1 \leq c \leq 1$ so that $f(c) = 0$.

c) for example, interval

$$[0.86, 0.87]$$

works since

$f(0.86) \approx 0.016$

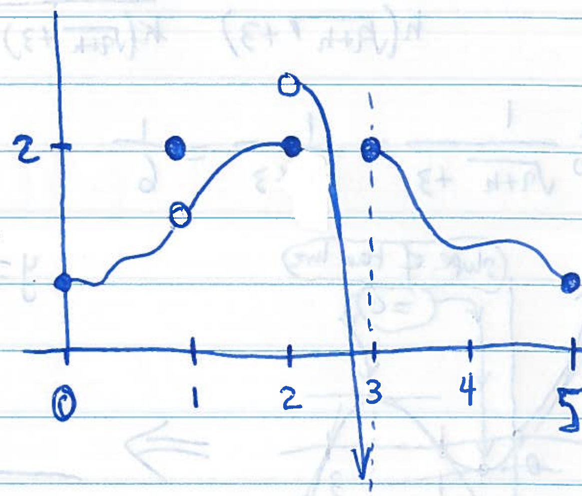
$f(0.87) \approx -0.013$

#150 The function h is NOT continuous at $x=2$, and so the intermediate value theorem does not hold!

#154 Places of discontinuity

$x=-1$	$x=0$	$x=1$
$\lim_{x\to -1} f(x)$ DNE	$f(0)$ does not exist AND $\lim_{x\to 0} f(x)$ DNE	$\lim_{x\to 1} f(x)$ exists $f(1)$ exists But they don't equal each other

#158



§3.1 #14

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{(1-h-h^2)-(1-0-0^2)}{h}$$

$$= \lim_{h\to 0} \frac{-h-h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-1-h)}{\cancel{h}} = \lim_{h\to 0} -1-h = -1$$

#16 $f'(1) = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{(1+h)+8} - \sqrt{1+8}}{h}$$

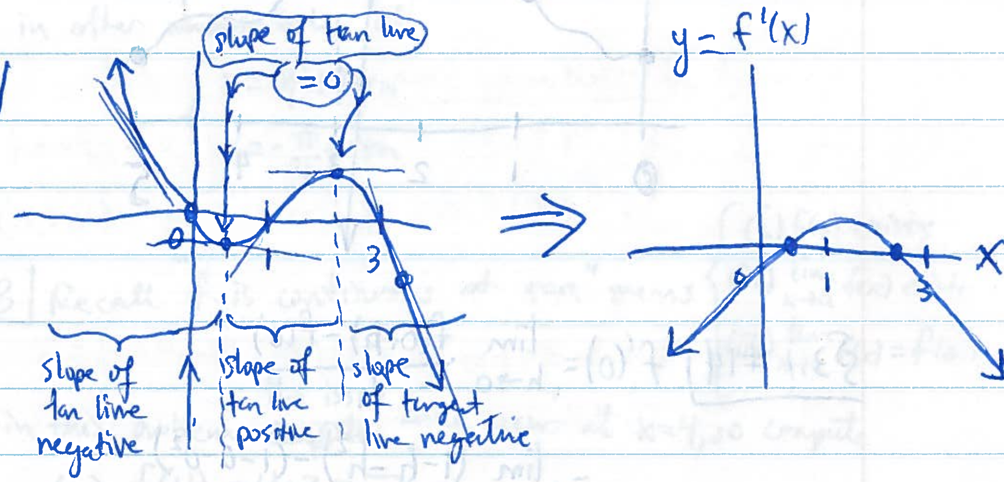
$$= \lim_{h\to 0} \underbrace{\frac{\sqrt{9+h} - 3}{h}}$$ $\to \frac{0}{0} \to$ more work to do

$$= \frac{\sqrt{9+h} - 3}{h}\left(\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right)$$

$$= \frac{9+h-9}{h(\sqrt{9+h}+3)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

#64



#90 a) average population between x years and x+h years

b) instantaneous rate of change of population with respect to years at year x