MATH 2501 Fall 2018 HW2

§2.2 #36| Guess $\lim_{x\to 0} \frac{\sin(3x)}{x}$ by filling out the table

Soln:

x	$\frac{\sin(3x)}{x}$
-0.1	2.95520
-0.01	2.9995500
-0.001	2.9999955
-0.0001	2.9999999955
0.0001	2.9999999955
0.001	2.9999955
0.01	2.9995500
0.1	2.95520

It appears that $\lim_{x\to 0} \frac{\sin(3x)}{x} = 3$.

#44| Investigate $\lim_{\theta\to 0} \sin\left(\frac{\pi}{\theta}\right)$

θ	$\sin\left(\frac{\pi}{\theta}\right)$
-0.1	0
-0.01	0
-0.001	0
-0.0001	0
0.0001	0
0.001	0
0.01	0
0.1	0

The function isn't always zero, so the table is misleading!

The graph looks like

↑ no limit at zero!!

#46 true

#48 false because

$f(-8) = -3$, but

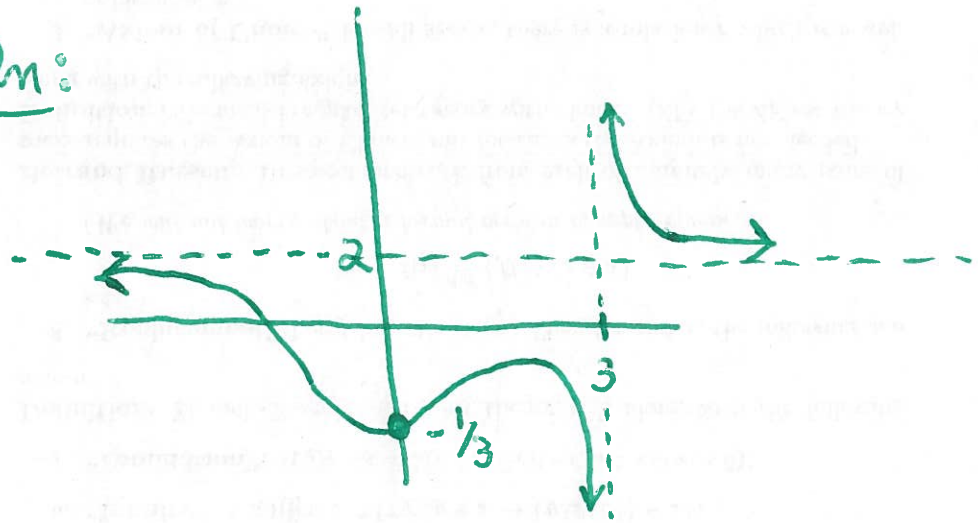
$\lim_{x \to -8} f(x) = -6$

#78 Draw a function that obeys

$\lim_{x \to -\infty} f(x) = 2$, $\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = \infty$

$\lim_{x \to \infty} f(x) = 2$, $f(0) = -\frac{1}{3}$

Soln:



§2.3

#92 $\lim_{x \to 3} \ln(e^{3x}) = \ln(e^9)$

$= 9$

#94 $\lim_{x \to 2} \frac{x-2}{x^2-2x}$ plug in $x=2$ $\frac{0}{0}$ ⇒ MORE WORK TO DO!

Algebra: $\frac{x-2}{x^2-2x} = \frac{(x-2)}{x(x-2)} = \frac{1}{x}$, so compute

$\lim_{x \to 2} \frac{x-2}{x^2-2x} = \lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$

↑ (algebra)

#96] $\lim_{h\to 0} \frac{(1+h)^2-1}{h}$ plug in $h=0$ $\frac{0}{0}$

Algebra: $\frac{(1+h)^2-1}{h} = \frac{1+2h+h^2-1}{h} = \frac{2h+h^2}{h} = \frac{h(2+h)}{h} = 2+h$;

so compute

$$\lim_{h\to 0} \frac{(1+h)^2-1}{h} = \lim_{h\to 0} 2+h = 2$$

(algebra)

#102] $\lim_{x\to -3} \frac{\sqrt{x+4}-1}{x+3}$ plug in $x=-3$ $\frac{0}{0}$

Algebra: $\frac{\sqrt{x+4}-1}{x+3} = \left(\frac{\sqrt{x+4}-1}{x+3}\right)\left(\frac{\sqrt{x+4}+1}{\sqrt{x+4}+1}\right)$ (the second factor $=1$)

$$= \frac{(x+4)-1}{(x+3)(\sqrt{x+4}+1)}$$

$$= \frac{(x+3)}{(x+3)(\sqrt{x+4}+1)} = \frac{1}{\sqrt{x+4}+1}$$

so compute

$$\lim_{x\to -3} \frac{\sqrt{x+4}-1}{x+3} = \lim_{x\to -3} \frac{1}{\sqrt{x+4}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

#112] Given $\lim_{x\to 6} h(x) = 6$, compute $\lim_{x\to 6} x\,h(x)$

Soln: Using limit laws,

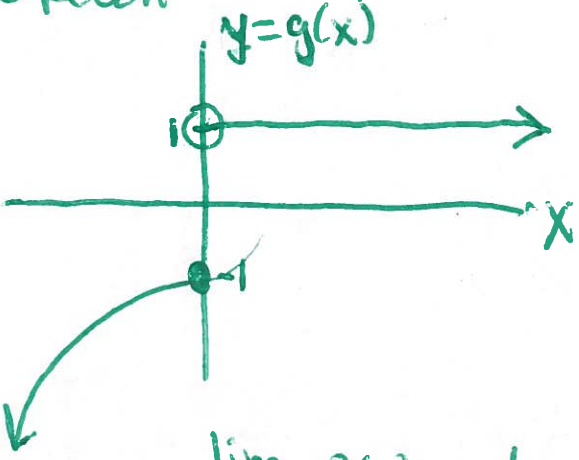
$$\lim_{x\to 6} x\,h(x) = \left(\lim_{x\to 6} x\right)\left(\lim_{x\to 6} h(x)\right)$$

$$= (6)(6) = 36$$

#116 Evaluate $\lim_{x\to 0^-} g(x)$ and $\lim_{x\to 0^+} g(x)$ for

$$g(x)=\begin{cases} x^3-1, & x\le 0 \\ 1, & x>0 \end{cases}$$

Soln: Sketch



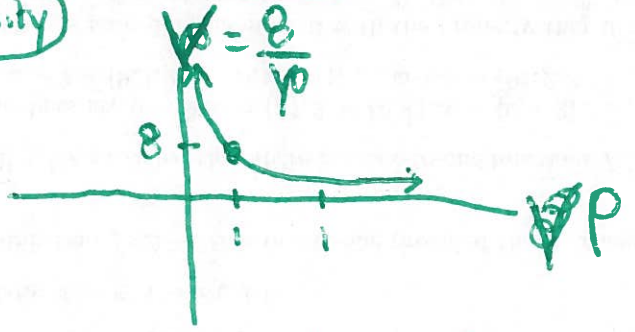
Compute $\lim_{x\to 0^-} g(x) = -1$ and

$\lim_{x\to 0^+} g(x) = 1$

#130 $\rho = \frac{m}{V}$ (mass / volume) $\Rightarrow V = \frac{m}{\rho}$

density

a)



b) $\lim_{\rho\to 0^+} V(\rho) = \lim_{\rho\to 0^+} \frac{m}{\rho} = +\infty$

"as density approaches zero of an object with mass m, the volume of the object increases to $+\infty$"