§1.1

#14 Find domain, range, and all intercepts of

$$f(x) = \frac{x}{x^2-16}$$

Solution: Here, domain is restricted by when denominator equals zero:

$$x^2-16=0 \Rightarrow x^2=16 \Rightarrow x=\pm\sqrt{16}=\pm 4$$

Thus domain is $\mathbb{R}\setminus\{4,-4\}$.

To find y-intercept, set $x=0$:

$$f(0)=\frac{0}{0^2-16}=0$$

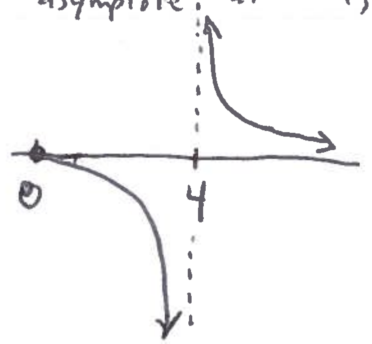
Thus, y-intercept is the point $(0,0)$.

To find x-intercept, set $f(x)=0$ + solve:

$$f(x)=0$$
$$\Downarrow$$
$$\frac{x}{x^2-16}=0 \Rightarrow x=0$$

Thus only x-intercept is at $(0,0)$.

Finally, the range is all of $\mathbb{R}$ because, for instance, near the vertical asymptote, at $x=4$, the graph of f looks like this:



← all y-values are touched by the graph

$\Downarrow$

range is $\mathbb{R}$

#17 Find domain, range, and all intercepts of
$$f(x) = -1 + \sqrt{x+2}$$

<u>Solution</u>: Here, domain is restricted by x-values which cause the stuff under the $\sqrt{\ }$ symbol to be negative, i.e.

$$x + 2 < 0 \implies \boxed{x < -2} \leftarrow \text{not in domain}$$

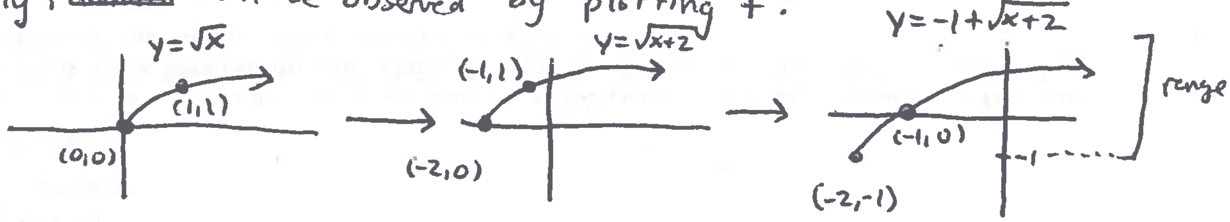
Thus domain is $\mathbb{R} \setminus (-\infty, -2)$, or also written as $[-2, \infty)$.

y-intercept comes from setting $x = 0$:
$$f(0) = -1 + \sqrt{0+2} = -1 + \sqrt{2}$$
thus y-intercept is the point $(0, -1+\sqrt{2})$

x-intercept comes from setting $f(x) = 0$:
$$f(x) = 0 \implies -1 + \sqrt{x+2} = 0$$
$$\implies \sqrt{x+2} = 1$$
square both sides $\rightarrow$
$$\implies x + 2 = 1$$
$$\implies \boxed{x = -1}$$

Thus x-intercept is at $(-1, f(-1)) = (-1, 0)$

Finally, ~~domain~~ range can be observed by plotting f:



Thus we see that the range is $[-1, \infty)$.

#40 For $f(x)=\sqrt{x}$, $g(x)=x-2$ find...

a) $f+g$

b) $f-g$

c) $f\cdot g$

d) $\frac{f}{g}$

and the domains of each.

Soln: a) $f+g$

$f(x)+g(x) = \sqrt{x} + x-2$

Domain of f: $[0,\infty)$

of g: $\mathbb{R}$

of $f+g$: $[0,\infty)$

b) $f-g$

$f(x)-g(x) = \sqrt{x} - (x-2) = \sqrt{x} - x + 2$

domain of $f-g$: $[0,\infty)$

c) $f\cdot g$

$f(x)g(x) = (\sqrt{x})(x-2) = x\sqrt{x} - 2\sqrt{x}$

domain of $fg = [0,\infty)$

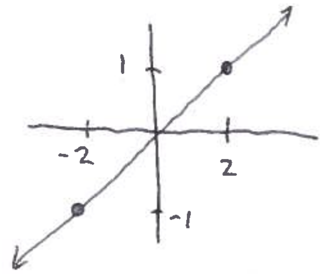
d) $\frac{f}{g}$

$\frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x-2} \longleftarrow$ $x=2$ not in domain b/c of division by zero

domain of $\frac{f}{g}$: $[0,\infty)\setminus\{2\} = [0,2)\cup(2,\infty)$

#71 Find an eqt of line passing thru $(2,1)$ and $(-2,-1)$

Soln:



$$\text{slope} = \frac{-1-1}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$$

Using point-slope formula

$$y - y_0 = m(x - x_0)$$

with point $(x_0, y_0) = (2,1)$ yields

$$\boxed{y - 1 = \frac{1}{2}(x-2)}$$

#84 For $f(x) = -3x^2 + 6x$ find...

a) degree

b) zeros

c) y-intercept

d) end behavior

Soln: a) the degree is 2 — the highest power on an x

b) find zeros by solving $f(x) = 0$:

$$-3x^2 + 6x = 0$$

$$x(-3x+6) = 0$$

$x = 0$ OR $-3x + 6 = 0 \Rightarrow -3x = -6$

$$\Rightarrow x = \frac{-6}{-3} = 2$$

c) y-intercept found by plugging in $x=0$: $f(0) = 0 \Rightarrow$ y-intercept is the point $(0,0)$

d) end behavior of even degree polynomial normally 
but since there is a minus sign on the degree 2 term,
the end behavior is 

§ 1.3

#155 Solve for $0 \le \theta \le 2\pi$

$$2\sin(\theta) - 1 = 0$$

Soln: Isolate $\sin(\theta)$ to get $\sin(\theta) = \frac{1}{2}$

Observe on unit circle that only angles for which the y-coordinate of the point is $\frac{1}{2}$ are at angles $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

§ 1.5

#272
$$\ln\left(a\sqrt[3]{b}\right) = \ln(a) + \ln(\sqrt[3]{b})$$
$$= \ln(a) + \ln(b^{1/3})$$
$$= \ln(a) + \frac{1}{3}\ln(b)$$

#277 Solve $e^{3x} - 15 = 0$

Soln: Add 15 to get

$$e^{3x} = 15$$

Plug both sides into $\ln(x)$ to get

$$\cancel{\ln(e}^{3x}) = \ln(15)$$

ln and e cancel

$$3x = \ln(15)$$

↓ divide by 3

$$x = \frac{\ln(15)}{3}$$

#288 Solve $\ln(\sqrt{x+3})=2$

Soln: Rewrite $\sqrt{\ }$ using fractional exponent:

$$\ln\left((x+3)^{1/2}\right)=2$$

$$\frac{1}{2}\ln(x+3)=2$$

$$\ln(x+3)=4$$

$\downarrow$ plug both sides into e^x

$$\cancel{e^{\ln}}{}^{(x+3)}=e^4$$

$$x+3=e^4$$

$$x=e^4-3$$

#307 A bacterial colony grown in a lab is known to double in number in 12 hours. Suppose, initially, there are 1000 bacteria present.

a) Use $Q=Q_0e^{kt}$ to determine the value k.

b) Determine approximately how long it takes for 200,000 bacteria to grow.

Soln: a) Here we are told $Q_0=1000$ and at time $t=12$, $Q=2000$. This yields

$$2000=1000e^{12k} \Rightarrow 2=e^{12k} \Rightarrow \ln(2)=12k$$

$$\Rightarrow k=\frac{\ln(2)}{12}\approx 0.0578$$

b) Here we want to find time t so that $Q=200{,}000$, i.e., solve

$$200000=1000e^{\frac{\ln(2)}{12}t} \Rightarrow 200=e^{\frac{\ln(2)}{12}t}$$

$$\Rightarrow \ln(200)=\frac{\ln(2)}{12}t$$

$$\Rightarrow t=\frac{12\ln(200)}{\ln(2)}\approx 91.72 \text{ hours}$$