Quiz 3 — MATH 1586 Spring 2018

1. Apply the ratio test to $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ to conclude whether or not the series converges.

   Solution: In this problem, $a_k = \frac{2^k}{k!}$. So, compute

$$
\begin{aligned}
\lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| &= \lim_{k\to\infty} \left| \frac{\frac{2^{k+1}}{(k+1)!}}{\frac{2^k}{k!}} \right| \\
&= \lim_{k\to\infty} \left| \frac{k!2^{k+1}}{2^k(k+1)!} \right| \\
&= \lim_{k\to\infty} \left| \frac{2}{k+1} \right| \\
&= 0.
\end{aligned}
$$

   Therefore, by the ratio test, since $0 < 1$, we conclude that the series converges.
   (*note: this series happens to converge to* $e^2 - 1$)

2. Apply the ratio test to $\sum_{k=1}^{\infty} 2^k$ to conclude whether or not the series converges.

   Solution: In this problem, $a_k = 2^k$. So, compute

$$
\begin{aligned}
\lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| &= \lim_{k\to\infty} \left| \frac{2^{k+1}}{2^k} \right| \\
&= \lim_{k\to\infty} |2| \\
&= 2.
\end{aligned}
$$

   Since $2 > 1$, we conclude by the ratio test that the series diverges.