

Quiz 11

Compute  $\text{curl } \vec{F}$  and  $\text{div } \vec{F}$  where  $\vec{F} = \langle x^2yz, xy^2z, xyz^2 \rangle$

Soln:

$$\text{curl } \vec{F} = \nabla \times \vec{F} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & xy^2z & xyz^2 \end{bmatrix}$$

$$= \vec{i} \det \begin{bmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2z & xyz^2 \end{bmatrix} - \vec{j} \det \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2yz & xyz^2 \end{bmatrix} + \vec{k} \det \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2yz & xy^2z \end{bmatrix}$$

$$= (xz^2 - xy^2)\vec{i} - (yz^2 - x^2y)\vec{j} + (y^2z - x^2z)\vec{k}$$

$$= \langle xz^2 - xy^2, x^2y - yz^2, y^2z - x^2z \rangle$$

$$\text{div } \vec{F} = \nabla \cdot \vec{F}$$

$$= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle x^2yz, xy^2z, xyz^2 \rangle$$

$$= 2xyz + 2xyz + 2xyz$$

$$= 6xyz$$