Quiz 1 — 25 January 2018

1. Calculate $\int x^2 + 5x + \frac{3}{x} \mathrm{d}x$.
   Solution: Compute

$$\begin{aligned}\int x^2 + 5x + \frac{3}{x} \mathrm{d}x &= \int x^2 \mathrm{d}x + \int 5x \mathrm{d}x + \int \frac{3}{x} \mathrm{d}x \\ &= \frac{x^3}{3} + \frac{5x^2}{2} + 3\ln(x) + C.\end{aligned}$$

2. Calculate $\int_5^6 e^{11t} \mathrm{d}t$.
   Solution: Let $u = 11t$ so that $\mathrm{d}u = 11\mathrm{d}t$. Thus $\frac{1}{11}\mathrm{d}u = \mathrm{d}t$. Now calculate

$$\begin{aligned}\int_5^6 e^{11t} \mathrm{d}t &\overset{u=11t}{=} \frac{1}{11} \int_{t=5}^{t=6} e^u \mathrm{d}u \\ &= \frac{1}{11} e^u \bigg|_{t=5}^{t=6} \\ &= \frac{1}{11} e^{11t} \bigg|_{t=5}^{t=6} \\ &= \left(\frac{1}{11} e^{66}\right) - \left(\frac{1}{11} e^{55}\right).\end{aligned}$$

3. The population of a city is projected to grow at a rate of

$$r(t) = 200 \left(1 + \frac{2t}{15 + t^2}\right) \frac{\text{people}}{\text{year}}.$$

   The current population (at time $t = 0$), t measured in years) is $25,000$. What will be the population in 4 years?
   Solution: To answer this, we must integrate $r(t)$ and then apply the initial condition. First, integrate: we will use the u-substitution $u = 15 + t^2$ so that $\mathrm{d}u = 2t\mathrm{d}t$ and now compute

$$\begin{aligned}\int r(t) \mathrm{d}t &= \int 200 \left(1 + \frac{2t}{15 + t^2}\right) \mathrm{d}t \\ &= 200 \int 1 \mathrm{d}t + 200 \int \frac{2t}{15 + t^2} \mathrm{d}t \\ &\overset{u=15+t^2}{=} 200t + 200 \int \frac{1}{u} \mathrm{d}u \\ &= 200t + 200\ln(u) + C \\ &= 200t + 200\ln(15 + t^2) + C.\end{aligned}$$

Find C

Now we apply the initial condition that, at $t = 0$, the value of this integral should be $25,000$:

$$\underbrace{25000}_{\text{given}} = \underbrace{200(0) + 200\ln(15 + 0^2) + C}_{\text{calculated}}$$

this yields

$$25000 = 200\ln(15) + C.$$

Therefore $C = 25000 - 200\ln(15) \approx 24458.39$. Thus the model for the population is

$$\text{population}(t) = 200t + 200\ln(15 + t^2) + 250000 - 200\ln(15).$$

In 4 years, i.e. at $t = 4$, the model predicts that the population will be

$$\text{population}(4) = 200(4)+200\ln(15+4^2)+25000-200\ln(15) \approx 25945 \text{ people.}$$

4. Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = 7y(t)$. Use the technique of separation of variables to solve the differential equation (your answer should have a "C" in it).

Solution: Separate variables to get

$$\frac{1}{y}\mathrm{d}y = 7\mathrm{d}t.$$

Place $\int$ on both sides of this equation to get

$$\int \frac{1}{y}\mathrm{d}y = \int 7\mathrm{d}t.$$

Finding these antiderivatives yields

$$\ln(y) = 7t + C.$$

Plug both sides of this equation into the exponential function to get

$$y = e^{7t+C}.$$