Some Problems — 24 January 2018

A *differential equation* is an equation (i.e. sentence containing a "=" sign) containing derivatives and an unknown function. The goal of a differential equation: find the unknown function.

1. ("Simple chemical conversion") From chemical experiments, it is known that for certain reactions where a substance converts into another substance, the rate of change (with respect to time) of the amount x at any time $t > 0$ is determined by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -kx(t),$$

for some constant k (we use "$-k$" in the equation because the amount of the substance is decreasing as it converts). The initial amount of the substance at time $t = 0$ is $x(t_0) = x_0$.

Suppose also that it is known (by measurement) that half of the substance converts by the end of 10 seconds.

At what time does $\dfrac{9}{10}$ of the substance become converted?

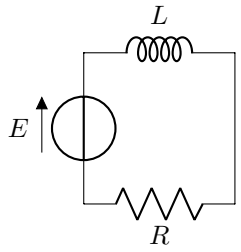
2. ("Newton's Law of Cooling") Newton's law of cooling states: the rate at which the temperature $T(t)$ changes in a cooling body is proportional to the difference between the temperature in the body and the constant temperature T_0 of the surrounding medium. In other words,

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(T - T_0),$$

where k is the proportionality constant and T_0 is the temperature of the surrounding medium.

When a cake is removed from a baking oven, its temperature is $300°F$. Three minutes later its temperature is $200°F$. How long will it take to cool off to room temperature in a room that is $70°F$?

3. ("RL-Circuit problem")



In a series circuit containing only a resistor (R, measured in ohms), an inductor (L, measured in henries which are ohm $\cdot$ sec), and a current at

time t, $i(t)$ (measured in amps), Kirchoff's second law states that the sum of the voltage drop across the inductor $\left(L\dfrac{\mathrm{d}i}{\mathrm{d}t}\right)$ and the voltage drop across the resistor (iR) is the same as the impressed voltage $(E(t))$ on the circuit. We obtain the differential equation for the current $i(t)$

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri(t) = E(t),$$

where L and R are constants known as the inductance and resistance respectively (note: the current $i(t)$ is sometimes called the *response* of the system).

A 30-volt electromotive force is applied to a series circuit in which the inductance is 0.1 henry and the resistance is 50 ohms. Find the current $i(t)$ if $i(0) = 0$.