Some Problems — 22 January 2018

1. The development of an amusement park on the outskirts of a city will increase the city's population at the rate of

$$4500\sqrt{t} + 1000 \ \frac{\text{people}}{\text{year}}$$

t years from the date of construction. The population before construction is $30,000$. Determine the projected population 9 years after construction of the park.

2. Nineteenth-century physician Jean Louis Marie Poiseuille discovered that the rate of change of the velocity of blood r cm from the central axis of an artery (in $\frac{\frac{\text{cm}}{\text{sec}}}{\text{cm}}$) is given by $a(r) = -kr$, where k is a constant. If the radius of an artery is R cm, find an expression for the velocity of blood as a function of r. Hint: $v'(r) = a(r)$ and $v(R) = 0$ (Why?)

3. The rate of change of coal exports from 2010 to 2012 was given by

$$f(t) = 31.863t^{-0.61} \ \frac{\text{million short tons}}{\text{year}}$$

for $0 \le t \le 2$, where t is measured in years with $t = 0$ corresponding to 2010. Find an expression for U.S. coal exports in year t. Assuming the trend continued through 2013, what were the U.S. coal exports that year?

4. The population of a certain city is projected to grow at the rate of

$$r(t) = 400\left(1 + \frac{2t}{24 + t^2}\right) \ \frac{\text{people}}{\text{year}}$$

for $0 \le t \le 5$. The current population is $60,000$. What will be the population 5 years from now?

5. A drug is carried into an organ of volume $V\text{cm}^3$ by a liquid that enters the organ at a rate if $a\frac{\text{cm}^3}{\text{sec}}$ and leaves it at the rate of $b\frac{\text{cm}^3}{\text{sec}}$. The concentration of the drug in the liquid entering the organ is $c\frac{\text{g}}{\text{cm}^3}$. If the concentration of the drug in the organ at time t (in seconds) is increasing at the rate of

$$x'(t) = \frac{1}{V}(ac - bx_0)e^{-\frac{bt}{V}} \ \frac{\frac{\text{g}}{\text{cm}^3}}{\text{sec}}$$

and the concentration of the drug in the organ initially is $x_0\frac{\text{g}}{\text{cm}^3}$, show that the concentration of the drug in the organ at time t is given by

$$x(t) = \frac{ac}{b} + \left(x_0 - \frac{ac}{b}\right)e^{-\frac{bt}{V}}.$$