Lab 8 — MATH 1586 Spring 2018

Recall: this site plots surfaces nicely:
http://web.monroecc.edu/manila/webfiles/pseeburger/CalcPlot3D/
Also, it is helpful to use WolframAlpha to compute integrals. You may also of course use Mathcad to do it.
Recall that if S is a surface parametrized by

$$\begin{cases} \vec{r}(u,v) \\ a \le u \le b \\ c \le v \le d, \end{cases}$$

then the vector field surface integral ("flux integral") is defined by

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{r} = \int_a^b \int_c^d \vec{F}\left(\vec{r}(u,v)\right) \cdot \left(\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right) \mathrm{d}t.$$

1. In the theory of electrostatics, Gauss' law says that the net charge enclosed by a closed surface S is

   $$Q = \epsilon_0 \iint_S \vec{E} \cdot \mathrm{d}\vec{S},$$

   where ϵ_0 is a constant called the "permittivity of free space".

   In this problem, you will find the flux (i.e. vector field surface integral) of the vector field $\vec{F} = \langle z, y, x\rangle$ across the unit sphere parametrized by $\vec{r}(u,v) = \langle \sin(v)\cos(u), \sin(v)\sin(u), \cos(v)\rangle$ for $0 \le u \le 2\pi$ and $0 \le \phi \le \pi$.

   (a) Draw the parametrized surface in this problem and attach an image of it to your lab.

   (b) Use WolframAlpha (or MathCad) to compute $\dfrac{\partial \vec{r}}{\partial u} \times \dfrac{\partial \vec{r}}{\partial v}$.

   (c) Write the vector field surface integral for this problem.

   (d) Use the computer to find the electric flux across the surface.

2. If the temperature at a point in space is $T(x,y,z)$ then the "heat flow" is defined as the vector field $\vec{F} = -K\nabla T$, where K is an experimentally derived constant called the "conductivity" of the substance. In this situation, the "heat flow across the surface S" is given by

   $$\iint_S \vec{F} \cdot d\vec{S} = -K \iint_S \nabla u \cdot d\vec{S}.$$

   Consider the metal cylinder parametrized as

   $$\begin{cases} \vec{r}(u,v) = \langle u, \sqrt{6}\cos(v), \sqrt{6}\sin(v)\rangle \\ 0 \le u \le 4 \\ 0 \le v \le 2\pi \end{cases}$$

   This cylinder has conductivity constant $K = 6.5$. The temperature T is $T(x,y,z) = 2y^2 + 2z^2$.

(a) Plot the surface and attach a picture of it.

(b) Find the heat flow vector field.

(c) Use WolframAlpha (or MathCad) to compute $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$.

(d) Write the vector field surface integral for this problem.

(e) Use the computer to find the rate of heat flow across the cylindrical surface.